In this work we explore the effects of higher twist power corrections on the deeply virtual Compton scattering process. The calculation of the helicity amplitudes for all possible polarization combinations is performed within the framework of QCD operator product expansion. As a result the known accuracy of the amplitudes is improved to include the (kinematic) twist-4 contributions. For the most part the analysis focuses on spin-1/2 targets, the answers for scalar targets conveniently emerge as a byproduct. We investigate the analytical structure of these corrections and prove consistency with QCD factorization. We give an estimation of the numerical impact of the sub-leading twist contributions for proton targets with the help of a phenomenological model for the nonperturbative proton generalized parton distributions. We compare different twist approximations and relate predictions for physical observables to experiments performed by the Hall A, CLAS, HERMES, H1 and ZEUS collaborations. The estimate also includes a numerical study for planned COMPASS-II runs. Throughout the analysis special emphasis is put on the convention dependence induced by finite twist truncation of scattering amplitudes.

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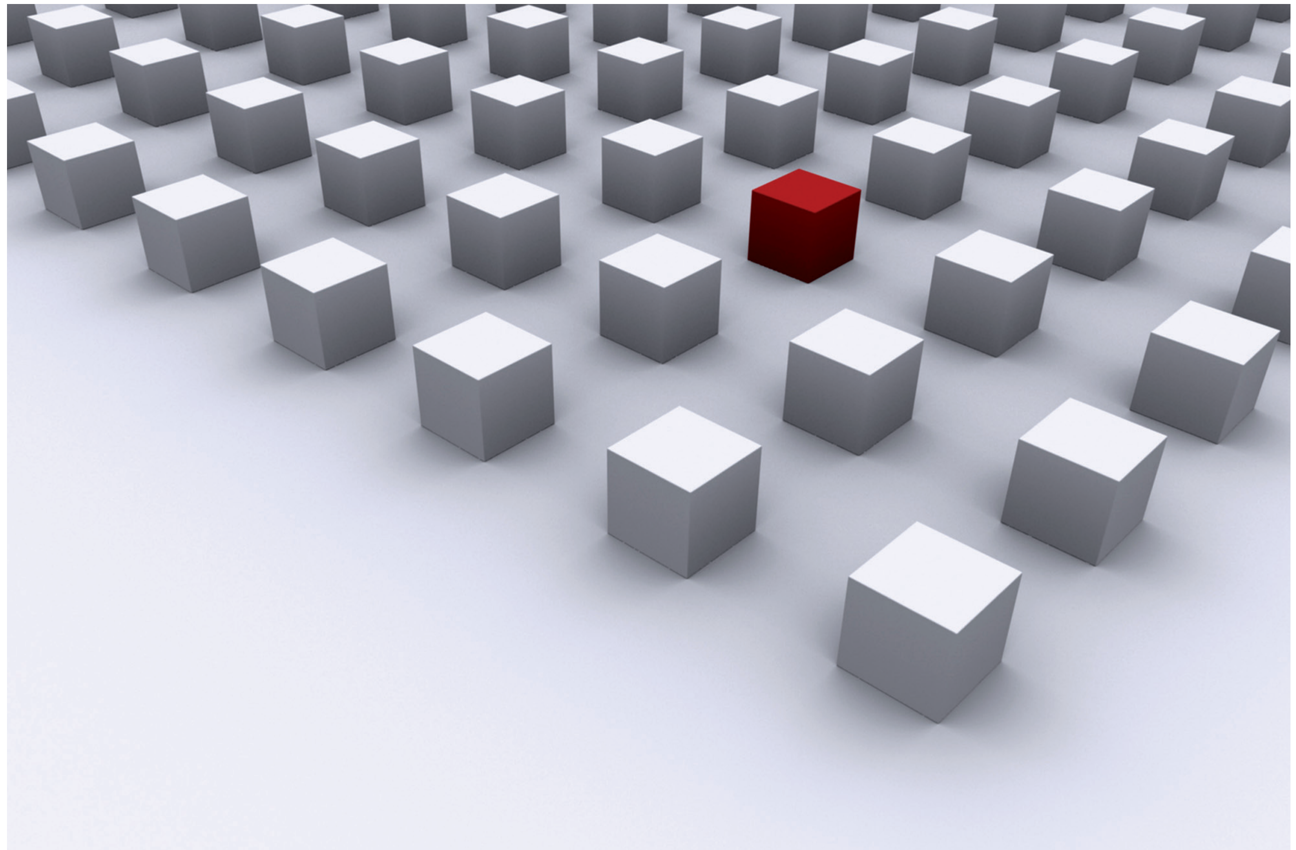

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Contents

1. Introduction

Understanding the internal structure of nucleons in terms of its underlying degrees of freedom – *quarks* and *gluons* – is a fundamental challenge to be addressed in quantum chromodynamics (QCD). An answer to this problem seems to be very elusive and today our picture is still far from complete.

Experimental activities on this subject began more than 50 years ago and are still ongoing. The cornerstone experiments started with the measurements by McAllister and Hofstadter [1], who determined the form factors of the proton and established its finite size. These findings triggered further interest in the substructure of the proton both from experimental and theoretical side. In early attempts, Gell-Mann and Zweig [2,3] proposed a model of quarks which build up hadronic matter. It had to be clarified if quarks were just a mathematical "trick" or actual particles. Progress came later through the first measurements on *deep inelastic scattering* (DIS) at SLAC [4,5], which pioneered further insight into the inner workings of a nucleon. The observed agreement with the *Bjorken scaling* [6] of the structure functions supported the mechanism that the scattering occurs off almost free point-like constituents, called *partons*. From today's theoretical perspective the partons are identified with the QCD building blocks, quarks and gluons. Violation of the Bjorken scaling behavior was predicted from QCD [7,8] and found later [9], providing an important test of the theory. DIS along with many other experiments established QCD as the accepted theoretical framework for hadronic matter.

Extracting predictions from QCD itself is in general a nontrivial problem. One reason for this is the intrinsic limitation of the applicability of perturbative methods. Perturbation theory works at high energies (or short distances) but it breaks down at small energies (long distances) through the running coupling constant $\alpha_{\mathrm{s}}(Q^2)$ [10,11]. Among the established nonperturbative approaches to the low energy sector of the theory, numerical methods from first principles based on lattice formulations of QCD seem to be the most promising. Most observables, e.g. cross sections, are a mixture of both long- and short-distance effects. A separation between the two domains is established by *factorization theorems*. Observables (in a general sense including amplitudes etc.) are written as products or convolutions of hard scattering coefficients, calculable in a perturbative framework, with phenomenological but universal nonperturbative functions. This approach necessarily introduces a factorization scale and the dependence of the two "factors" on it can be studied by renormalization group methods or evolution equations. The driving evolution kernels can be calculated order by order in perturbation theory. The nonperturbative input is a priori unknown, but one can revert to a phenomenological treatment and extract it from experimental data, usually by means of a suitable model or parametrization. In the case of DIS these are the *parton distribution functions* (PDFs), describing the probability densities to find a certain parton with a given longitudinal momentum and polarization inside a fast moving nucleon. Universality guarantees that, once determined from one set of measurements, the PDFs can be used to describe any other experiment to which they contribute.

More rigorously, in the sense of a quantum field theory, the PDFs are defined by forward matrix elements of nonlocal twist-2 light-ray operators bilinear in quark or gluon fields. Such PDFs contain some, but certainly not all information about the nucleon substructure. They are, by definition, restricted to the longitudinal degree of freedom and insensitive to multi-particle correlations. PDFs should rather be regarded as one of many aspects of the nucleon landscape, determining its global shape. In principle, a complete "nucleon map", would require the knowledge of the whole nucleon wave function, including infinitely many Fock components. At present, a theoretical solution to this bound state problem in QCD is out of reach. Instead one tries to view the nucleon from different "angles" by considering generalizations of PDFs or operators that encode more or different feedback from the nucleon. Necessarily there should be at least one experiment sensitive to such extensions.

One of the most prominent concepts that emerged in the last two decades is that of *generalized parton distributions* (GPDs), defined in [12–14]. Excellent reviews on the subject can be found in Refs. [15,16]. The distinction from the ordinary PDFs is simple: instead of taking the forward matrix element of the light-ray operators, say $\langle p|\dots|p\rangle$, one considers the off-forward one, $\langle p'|\dots|p\rangle$, with two, possibly different, momentum eigenstates of the hadron. In addition to the usual PDF variables x and Q^2 GPDs now depend on two more variables, the squared *momentum transfer* $t = (p - p')^2$ and the *skewness* ξ, which is essentially a longitudinal fraction of the momentum difference. This innocent-looking generalization opens a wide door into the nucleon landscape. Two of the "hot topics" that can be accessed within GPDs shall not go unmentioned. Already in early developments, Ji [17] realized that certain moments of GPDs are related to the nucleon energy momentum tensor (or better yet a particular version of it). Thus they can be used to quantify how the spin and orbital angular momentum is distributed among quarks and gluons. There are several subtleties about this decomposition, being actively debated to the present day, see Ref. [18] for a recent review. Apart from that, it was shown later [19,20], that GPDs provide a valuable source of information about the momentum distribution of partons, that goes beyond the usual collinear PDF description and allows one to access a three-dimensional spatial image of the nucleon. Through the so-called *impact parameter representations*, GPDs encode a probabilistic distribution of partons in the plane transverse to the nucleon's direction of motion (in the infinite momentum frame) as a function of the distance from the nucleon's center. The picture, which was originally formulated at zero skewness, was refined later [21] and shown to hold also for nonzero ξ (with a change of the center of transverse momentum interpretation).

Constraints on GPDs arise through the observation that some of the "old" key observables in hadron physics are naturally contained in them. Most importantly, DIS constrains those GPDs, which allow a reduction to the usual PDFs in the limit $\xi \to 0$, $t \to 0$ or $p' \to p$. Further requirements come from certain Mellin moments in x, the first moments being related to the elastic form factors and the anomalous magnetic moment. The second moments enter directly in Ji's spin sum rule [17].

Having established the physical significance of GPDs, one realizes that they are probed, apart from the aforementioned limiting cases, in hard exclusive reactions with nonzero momentum transfer. The bulk of experimental data comes from *deeply virtual meson production* and *deeply virtual Compton scattering* (DVCS). The latter, which is also the main topic of this work, is defined as the process where a photon of high virtuality Q^2 scatters off a nucleon with emission of a real photon in the final state. It appears as part of the leptoproduction of a photon off a nucleon and is regarded as the cleanest reaction channel to access GPDs. Factorization for this process has been proven in the limit of large Q^2 [22]. In reality many of the existing and future measurements lie somewhere in the ballpark of $Q^2 \sim 2 - 15$ GeV^2. Naturally, an analysis of the subleading corrections in $1/Q$ is important.

The theoretical description for DVCS relies on the *operator product expansion* (OPE) of the time-ordered product of two electromagnetic quark currents. Suppressed contributions originate from higher twist operators in this framework. Here we focus on a particular subset of $1/Q$-effects, dubbed *kinematic power corrections* [23,24]. They are analogous to the so-called *Nachtmann corrections* [25] in DIS, which stem from the "subtraction of traces" prescription for the leading twist operators. For DVCS one faces additional complications since the contributions of total derivatives of the twist-2 operators have to be included as well. In DIS they are absent, since matrix elements of total derivatives are proportional to the momentum differences of the initial and final state. The technically demanding operators are those of the form $(\partial\mathcal{O}) = \partial^\mu \mathcal{O}_{\mu\mu_1\ldots\mu_n}$, where $\mathcal{O}_{\mu\mu_1\ldots\mu_n}$ is a local (conformal) quark-antiquark leading twist operator. Due to a theorem by Ferrara et al. [26] $(\partial\mathcal{O})$ vanishes in the free field theory. By QCD equations of motion in the interacting theory one can relate $(\partial\mathcal{O})$ to three-particle quark-antiquark-gluon operators, which appear (among others) in the OPE at twist-4 level. The separation of contributions proportional to $(\partial\mathcal{O})$ from the OPE is a very involved algebraic task and has been solved only recently [23,24]. Parts of this thesis are based on these results. In the kinematic approximation one considers only the leading twist descendants and neglects the "genuine" multi-particle correlations in the target, i.e. those that are not related to the leading twist operators by QCD equations of motion. By definition this does not introduce any (new) nonperturbative input apart from the GPDs themselves. As a consequence we are able to compute the DVCS process amplitudes including mass (m^2/Q^2) and momentum transfer (t/Q^2) corrections. The latter are of particular relevance, given the fact that for the three-dimensional imaging of the nucleon, a sufficiently broad interval in $|t|$, maybe up to 2 GeV^2 [27], needs to be covered by experiments. Note that it is a priori not clear whether factorization still holds for the power corrections and we shall address this question.

The presentation is organized as follows: In the next chapter we briefly spell out our conventions and necessary notations. Chapter 3 reviews key features of the operator product expansion to (kinematic) twist-4 accuracy, in particular relevant contributions to off-forward reactions. The prime reaction of interest, deeply virtual Compton scattering, is introduced in Chapter 4 along with its kinematics, amplitudes and the ever present generalized parton distributions and their parametrizations in terms of double distributions. The latter form a convenient foundation for the calculation of helicity amplitudes. We outline technical details and intermediate expressions in Chapter 5. Further processing of the results is presented in Chapter 6, investigating their properties and giving several equivalent representations. By selecting a popular GPD model, the phenomenological impact is examined in Chapter 7 through a comparison with leading twist conventions and available experimental data on several representative observables. Finally we conclude in Chapter 8 and outline further possible applications. In addition we include the Appendices A, B and C, where technical questions of general relevance for this work are addressed.

2. Conventions and light-cone formalism

For this work it is of utmost importance to specify the conventions of special relativity. We devote this chapter to a detailed and hopefully unambiguous presentation of the notation. It may serve as a reference for future applications.

Our choice of the metric tensor of the Minkowski space has a "mostly negative" signature,

$$(g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{2.1}$$

For the γ-matrices we use the *Weyl representation*,

$$\gamma^0 = \gamma_0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} , \quad \gamma^i = -\gamma_i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} , \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} , \tag{2.2}$$

where σ^i , $i = 1, 2, 3$ are the three Pauli matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{2.3}$$

The definition of γ_5 can equivalently be written as

$$\gamma_5 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma , \quad \varepsilon_{0123} = 1 , \tag{2.4}$$

occasionally known as the *Bjorken-Drell convention* [28].

With the form (2.2) of the γ-matrices the generators of the Lorentz group

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \tag{2.5}$$

are 2×2 block-diagonal. This already implies that a Dirac spinor q does not transform according to an irreducible representation of the Lorentz group. The 2×2 blocks cannot be diagonalized further, essentially because the Pauli matrices do not admit it. Therefore a Dirac spinor is an element of the direct sum of two irreducible representations, which are often dubbed $(1/2, 0)$ and $(0, 1/2)$, each of dimension two. By Hermitian conjugation of $(1/2, 0)$ one gets a representation that is equivalent to $(0, 1/2)$. We write a Dirac spinor q as

$$q = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} , \tag{2.6}$$

where the two-component objects ψ_α, $\bar{\chi}^{\dot{\alpha}}$, which transform according to an irreducible representation of the Lorentz group, are called *Weyl spinors*. They correspond to the left- and right-handed projections of q respectively. To distinguish the respective chirality, we use dotted and undotted indices for the components of the Weyl spinors. The Dirac-adjoint spinor $\bar{q} = q^\dagger \gamma_0$ is

$$\bar{q} = \left(\chi^\alpha, \bar{\psi}_{\dot\alpha}\right), \quad \chi^\alpha = \left(\bar{\chi}^{\dot\alpha}\right)^\dagger, \quad \bar{\psi}_{\dot\alpha} = \left(\psi_\alpha\right)^\dagger. \tag{2.7}$$

In that context it is necessary to define raising and lowering of spinor indices, which is done with the help of the two-dimensional Levi-Civita symbol

$$u^\alpha = \varepsilon^{\alpha\beta} u_\beta, \quad u_\alpha = \varepsilon_{\beta\alpha} u^\beta, \quad \bar{u}^{\dot\alpha} = \varepsilon^{\dot\beta\dot\alpha} \bar{u}_{\dot\beta}, \quad \bar{u}_{\dot\alpha} = \varepsilon_{\dot\alpha\dot\beta} \bar{u}^{\dot\beta}, \tag{2.8}$$

for arbitrary Weyl spinors $u_\alpha, \bar{u}_{\dot\alpha}$. The convention for ε is as follows:

$$\begin{aligned} &\varepsilon_{12} = \varepsilon^{12} = 1, \quad \varepsilon_{21} = \varepsilon^{21} = -1, \\ &\varepsilon_{\dot2\dot1} = \varepsilon^{\dot2\dot1} = 1, \quad \varepsilon_{\dot1\dot2} = \varepsilon^{\dot1\dot2} = -1. \end{aligned} \tag{2.9}$$

Note that indeed Eqs. (2.8), (2.9) guarantee that raising followed by lowering of an index (or vice versa) is the identity operation. Quite frequently one encounters the contraction of two Weyl spinors, for which we introduce a shorthand notation, The convention adopted here follows an "up-down" rule for undotted and a "down-up" rule for dotted indices, i.e.

$$(uv) = u^\alpha v_\alpha, \quad (\bar{u}\bar{v}) = \bar{u}_{\dot\alpha} \bar{v}^{\dot\alpha}, \tag{2.10}$$

which is different in sign compared to the reversed case, i.e.

$$(vu) = -(uv), \quad (\bar{v}\bar{u}) = -(\bar{u}\bar{v}). \tag{2.11}$$

As a trivial example consider the scalar combination $\bar{q}q$ where q and $\bar{q}$ are the Dirac spinor and its adjoint, then

$$\bar{q}q = (\chi\psi) + (\bar{\psi}\bar{\chi}). \tag{2.12}$$

The usual Lorentz vectors are also incorporated in this formalism by the following construction: One uses the 2×2 unit matrix $\mathbb{1}$ and the three Pauli matrices to map the vector x_μ to 2×2 matrices $(x) = x_\mu \sigma^\mu$ with $\sigma^\mu = (\mathbb{1}, \vec\sigma)$ and $(\bar{x}) = x_\mu \bar\sigma^\mu$ with $\bar\sigma^\mu = (\mathbb{1}, -\vec\sigma)$, which take the form

$$\begin{aligned} \left(x_{\alpha\dot\alpha}\right) &= x_\mu \left(\sigma^\mu\right)_{\alpha\dot\alpha} = \begin{pmatrix} x_0 + x_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 - x_3 \end{pmatrix}, \\ \left(\bar{x}^{\dot\alpha\alpha}\right) &= x_\mu \left(\bar\sigma^\mu\right)^{\dot\alpha\alpha} = \begin{pmatrix} x_0 - x_3 & -x_1 + ix_2 \\ -x_1 - ix_2 & x_0 + x_3 \end{pmatrix}. \end{aligned} \tag{2.13}$$

The "first" index always refers to the row and the second index to the column of the matrix, e.g. $x_{1\dot2} = x_1 - ix_2 = -\bar{x}^{\dot12}$. In the conventional light-cone formalism the diagonal entries correspond to the "plus" and "minus" projections on the light-cone. The components transverse to it are encoded in the off-diagonal holomorphic and anti-holomorphic (in x_1, x_2) entries. If x_μ is a real-valued vector, then the matrices $(x_{\alpha\dot\alpha})$ and $(\bar{x}^{\dot\alpha\alpha})$ are Hermitian. The Minkowski scalar product of two vectors x_μ and y_μ can be written as half of the trace of the product of one "barred" and one "unbarred" matrix

$$(xy) = \frac{1}{2} x_{\alpha\dot\alpha} \bar{y}^{\dot\alpha\alpha}. \tag{2.14}$$

Raising and lowering of spinor indices works just as in the spinor case, e.g.

$$x^{\alpha\dot{\alpha}} = \varepsilon^{\alpha\beta} x_{\beta\dot{\beta}} \varepsilon^{\dot{\beta}\dot{\alpha}} = \bar{x}^{\dot{\alpha}\alpha} \,. \tag{2.15}$$

In practical calculations the Fierz identities for Pauli-matrices come in handy:

$$(\sigma^\mu)_{\alpha\dot{\alpha}} (\bar{\sigma}_\mu)^{\dot{\beta}\beta} = 2\delta_\alpha^\beta \delta_{\dot{\alpha}}^{\dot{\beta}} \,, \quad (\sigma^\mu)_{\alpha\dot{\alpha}} (\sigma_\mu)_{\beta\dot{\beta}} = -2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} \,. \tag{2.16}$$

If x_μ is a real-valued Lorentz vector, then its associated matrix obeys

$$(x^\dagger)_{\alpha\dot{\alpha}} = \bar{x}_{\dot{\alpha}\alpha} \,. \tag{2.17}$$

Note that the above formulation for vectors generalizes trivially to any Lorentz tensor, i.e. a Lorentz tensor of rank n, say $t_{\mu_1\cdots\mu_n}$, is mapped to a spinorial tensor with n dotted and n undotted indices,

$$t_{\alpha_1\dot{\alpha}_1\cdots\alpha_n\dot{\alpha}_n} \equiv \sigma^{\mu_1}_{\alpha_1\dot{\alpha}_1} \cdots \sigma^{\mu_n}_{\alpha_n\dot{\alpha}_n} t_{\mu_1\cdots\mu_n} \,. \tag{2.18}$$

A nice collection of helpful formulas on this particular topic is provided in Ref. [29], however one should keep in mind that [29] uses partially different conventions compared to this work.

In some situations it is useful to have explicit representations for the solutions of the free Dirac equation $(\not{p} - m)u_\lambda(p) = 0$, $(\not{p} + m)v_\lambda(p) = 0$ with mass m and helicity λ. Our choice corresponds to [30]

$$\begin{aligned} u_\uparrow(p) &= \frac{1}{\sqrt{p^0+p^3}} \begin{pmatrix} m \\ 0 \\ p^0+p^3 \\ p^1+ip^2 \end{pmatrix}, \quad u_\downarrow(p) = \frac{1}{\sqrt{p^0+p^3}} \begin{pmatrix} ip^2-p^1 \\ p^0+p^3 \\ 0 \\ m \end{pmatrix}, \\ v_\uparrow(p) &= \frac{1}{\sqrt{p^0+p^3}} \begin{pmatrix} ip^2-p^1 \\ p^0+p^3 \\ 0 \\ -m \end{pmatrix}, \quad v_\downarrow(p) = \frac{1}{\sqrt{p^0+p^3}} \begin{pmatrix} -m \\ 0 \\ p^0+p^3 \\ p^1+ip^2 \end{pmatrix}, \end{aligned} \tag{2.19}$$

and implies the normalization conditions $\bar{u}_\lambda(p)u_{\lambda'}(p) = 2m\delta_{\lambda\lambda'}$, $\bar{v}_\lambda(p)v_{\lambda'}(p) = -2m\delta_{\lambda\lambda'}$. The helicity labels refer to the eigenvalues of the spin projection along the momentum for a particle moving fast in the negative z-direction. In more detail, the helicity operator h_∞ in this infinite momentum frame reads [30]

$$h_\infty(p) = \frac{1}{2} \begin{pmatrix} 1 & 2\frac{p^1-ip^2}{p^0+p^3} & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2\frac{p^1+ip^2}{p^0+p^3} & -1 \end{pmatrix}, \tag{2.20}$$

and is diagonalized by $u_{\uparrow,\downarrow}(p)$, $v_{\uparrow,\downarrow}(p)$,

$$\begin{aligned} h_\infty(p)u_\uparrow(p) &= +\frac{1}{2}u_\uparrow(p) \,, \quad h_\infty(p)u_\downarrow(p) = -\frac{1}{2}u_\downarrow(p) \,, \\ h_\infty(p)v_\uparrow(p) &= -\frac{1}{2}v_\uparrow(p) \,, \quad h_\infty(p)v_\downarrow(p) = +\frac{1}{2}v_\downarrow(p) \,. \end{aligned} \tag{2.21}$$

3. Aspects of the operator product expansion

3.1. Formulation of the problem

In many physical applications where some hadronic system is probed by electromagnetic interactions, observables are parametrized in terms of products of electromagnetic currents constructed from quark fields. Examples include the e^+e^- annihilation into hadrons, the famous deep inelastic scattering and, most relevant here, Compton scattering. For our purposes the goal is to study the behavior of the time-ordered product of two currents

$$T_{\mu\nu} = i\,\mathrm{T}\big\{j_\mu(z_1 x) j_\nu(z_2 x)\big\}\,. \tag{3.1}$$

Here z_1 and z_2 are some real numbers, their role will be discussed later. The current is defined as

$$j_\mu(x) = \bar{q}(x)\gamma_\mu q(x)\,. \tag{3.2}$$

For simplicity we consider just one quark flavor, and the summation over equal colors, $\bar{q}(x)\gamma_\mu q(x) \equiv \bar{q}_i(x)\gamma_\mu q_i(x)$, is left implicit. Hard processes are dominated from regions of $T_{\mu\nu}$ near the light-cone $x^2 \to 0$. A technical complication arises since the product of fields at light-like distances is ill-defined in general due to a singular behavior in $1/x^2$. Simple examples can be constructed e.g. in the free field theory. To make this more tractable Wilson [31] proposed a Laurent-like series for products of fields with possibly singular coefficient functions and regular operators, called operator product expansion. A formal proof of the OPE was given later by Zimmermann [32].

Following [33], let us review heuristically how the OPE works in practice. The T-product consists of four fermionic operators. According to the Wick theorem, one can rewrite it in terms of all possible products of contracted $q\bar{q}$ fields with the remaining uncontracted fields in normal order. For the scattering processes which we are going to consider, we can ignore disconnected Feynman diagrams. The contributions where all fields are contracted would correspond to one of those and can be disregarded for our purposes. The leading singular (in $1/x$) terms are extracted by contracting two of the quark fields and leaving the remaining two uncontracted:

$$T_{\mu\nu} = i\big(\bar{q}(z_1x)\gamma_\mu \overbracket{q(z_1x)\bar{q}}(z_2x)\gamma_\nu q(z_2x) + \overbracket{\bar{q}(z_1x)\gamma_\mu q(z_1x)\bar{q}(z_2x)\gamma_\nu q}(z_2x) + \dots\big)\,. \tag{3.3}$$

A contribution of all fields left uncontracted also exists, but it is not singular as $x^2 \to 0$ and can be neglected. In the leading order the contraction $\overbracket{q(z_1x)\bar{q}}(z_2x)$ is the (massless) propagator of a quark in coordinate representation

$$\overbracket{q(z_1x)\bar{q}}(z_2x) = \frac{i\not{x}}{2\pi^2 (z_{12})^3 (x^2 - i0)^2} + \dots\,. \tag{3.4}$$

Again the notation is kept short here, suppressing explicit Dirac indices and a unit matrix in color space. In the above formula, the "$i0$" stands for "i times an infinitesimal positive number" and corresponds to Feynman's causality prescription. In the following we will leave it implicit, whenever its appearance is unimportant. We also use the following abbreviation

$$z_{12} = z_1 - z_2\,. \tag{3.5}$$

Including the correction to the first order in the coupling g one gets, cf. [33]

$$\overbracket{q(z_1x)\bar{q}}(z_2x) = \frac{i\not{x}}{2\pi^2(z_{12})^3x^4} + \int d^4y\,\frac{i(z_1\not{x}-\not{y})}{2\pi^2(z_1x-y)^4}ig\not{A}(y)\frac{i(\not{y}-z_2\not{x})}{2\pi^2(y-z_2x)^4} + \dots\,. \tag{3.6}$$

The color structure in the second term is contained in the field $(\not{A}(y))_{ij} \equiv T^a_{ij}\not{A}^a(y)$, when i and j are the color indices of $q(z_1x)$ and $\bar{q}(z_2x)$ respectively. T^a_{ij} are the generators of the color gauge group SU(3) in the fundamental representation with $i,j \in \{1,2,3\}$ and $a \in \{1,\dots 8\}$. To make progress on this term

$$\Delta G(z_1x, z_2x) \equiv \int d^4y\,\frac{i(z_1\not{x}-\not{y})}{2\pi^2(z_1x-y)^4}ig\not{A}(y)\frac{i(\not{y}-z_2\not{x})}{2\pi^2(y-z_2x)^4} \tag{3.7}$$

we introduce the Feynman parametrization to combine the two denominators and shift the y-integration by $y \to y + z^u_{21}x$ to cast it into the form

$$\Delta G(z_1x, z_2x) = -ig\frac{3}{2\pi^4}\int_0^1 du\,u\bar{u}\int d^4y\,\frac{(z_{12}\bar{u}\not{x}-\not{y})\not{A}(y+z^u_{21}x)(\not{y}+z_{12}u\not{x})}{[y^2+u\bar{u}(z_{12}x)^2]^4}\,, \tag{3.8}$$

where

$$z^u_{21} = \bar{u}z_2 + uz_1\,, \quad \bar{u} = 1-u\,. \tag{3.9}$$

Since we want to study the behavior of $T_{\mu\nu}$ at small distances x, we expand the gauge field around $y = 0$,

$$\not{A}(y+z^u_{21}x) = \not{A}(z^u_{21}x) + \big[(y\partial)\not{A}\big](z^u_{21}x) + \dots\,, \tag{3.10}$$

from which we obtain

$$\begin{aligned}\Delta G(z_1x, z_2x) = &-g\frac{\not{x}}{2\pi^2(z_{12})^2x^4}\int_0^1 du\,(xA)(z^u_{21}x)\\ &-\frac{g}{8\pi^2z_{12}x^2}\int_0^1 du\,(\bar{u}\not{x}\gamma_\mu\gamma_\nu - u\gamma_\nu\gamma_\mu\not{x})(\partial^\nu A^\mu)(z^u_{21}x) + \dots\,.\end{aligned} \tag{3.11}$$

Here the ellipses stand for terms of higher order in x. In deriving Eq. (3.11), we utilized the integrals

$$\begin{aligned}\int d^4y\,\frac{1}{(-y^2+z^2+i0)^n} &= -i\pi^2\frac{\Gamma(n-2)}{\Gamma(n)}\frac{1}{(z^2+i0)^{n-2}}\,,\\ \int d^4y\,\frac{y_\mu y_\nu}{(-y^2+z^2+i0)^n} &= +i\pi^2\frac{\Gamma(n-3)}{2\Gamma(n)}\frac{g_{\mu\nu}}{(z^2+i0)^{n-3}}\,.\end{aligned} \tag{3.12}$$

Note that in the Taylor expansion of $A(y+z^u_{21}x)$, Eq. (3.10), higher order terms in y would always produce suppressed, less singular, contributions in x. The first term in Eq. (3.11) together with the leading order expression forms the first order in g of the *path-ordered exponential*,

$$
\begin{aligned}
[z_1 x, z_2 x] &= \mathrm{Pexp}\Big(ig \int_0^1 du\, z_{12} (xA)(z_{21}^u x) \Big) \\
&\equiv \sum_{n=0}^{\infty} (i z_{12} g)^n \int_0^1 du_1 \int_0^{u_1} du_2 \cdots \int_0^{u_{n-1}} du_n\, (xA)(z_{21}^{u_1} x) \cdots (xA)(z_{21}^{u_n} x)\,.
\end{aligned}
\tag{3.13}
$$

Its role is to restore gauge invariance for the remaining fields at different points. Note that the order of the gauge fields in (3.13) is important, since A is matrix-valued.

For simplicity we employ the Fock-Schwinger gauge, which is defined by the condition

$$
(xA)(x) = 0\,, \quad A_\mu(0) = 0\,. \tag{3.14}
$$

From these two requirements follows a simple integral representation of the gauge field A_μ in terms of the field strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^a_\mu A^b_\nu$ (the structure constants of SU(3) are $f^{abc} = -2i\,\mathrm{Tr}\big([T^a, T^b] T^c\big)$),

$$
A_\mu(x) = \int_0^1 d\alpha\, \alpha x^\nu F_{\nu\mu}(\alpha x)\,, \quad A_\mu = A^a_\mu T^a\,, \quad F_{\mu\nu} = F^a_{\mu\nu} T^a\,. \tag{3.15}
$$

In this gauge the path-ordered exponential reduces to unity. Further, by using the above formula and the equation of motion, the total divergence (∂A) is of order g. This, along with the γ-matrix identity

$$
\gamma_\alpha \gamma_\mu \gamma_\nu = g_{\alpha\mu}\gamma_\nu + g_{\mu\nu}\gamma_\alpha - g_{\alpha\nu}\gamma_\mu - i\gamma_5 \gamma^\rho \varepsilon_{\rho\alpha\mu\nu} \tag{3.16}
$$

allows us to rewrite our expression for the propagator in terms of $F_{\mu\nu}$ and $\widetilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$,

$$
\overbracket{q(z_1 x)\bar{q}}(z_2 x) = \frac{i\not{x}}{2\pi^2 (z_{12})^3 x^4} + \frac{g}{8\pi^2 z_{12} x^2} \int_0^1 du\, x_\mu \gamma_\nu \big[(\bar{u} - u) F^{\mu\nu}(z_{21}^u x) - i\gamma_5 \widetilde{F}^{\mu\nu}(z_{21}^u x) \big]\,, \tag{3.17}
$$

up to higher order corrections in x and g. To the same accuracy one readily reads off the expansion of the time-ordered product, Eq. (3.3):

$$
T_{\mu\nu} = T^{(a)}_{\mu\nu} + T^{(b)}_{\mu\nu} + \big(\mu \leftrightarrow \nu, z_1 \leftrightarrow z_2\big)\,, \tag{3.18}
$$

with

$$
\begin{aligned}
T^{(a)}_{\mu\nu} &= -\frac{1}{2\pi^2 (z_{12})^3 x^4} \bar{q}(z_1) \gamma_\mu \not{x} \gamma_\nu q(z_2)\,, \\
T^{(b)}_{\mu\nu} &= \frac{g}{8\pi^2 z_{12} x^2} \int_0^1 du\, \bar{q}(z_1) \gamma_\mu x_\rho \gamma_\sigma \big[i(\bar{u} - u) F^{\rho\sigma}(z_{21}^u) + \gamma_5 \widetilde{F}^{\rho\sigma}(z_{21}^u) \big] \gamma_\nu q(z_2)\,,
\end{aligned}
\tag{3.19}
$$

where we have introduced a shorthand notation for the arguments of the fields,

$$
\bar{q}(z_1) \equiv \bar{q}(z_1 x) \text{ etc.,} \tag{3.20}
$$

leaving the dependence on the space-time point x implicit. A manifestly gauge invariant form of Eqs. (3.19) is obtained by restoring the proper gauge links between the fields. The expressions (3.19) can be viewed as the starting point of the calculation in Ref. [23]. In order to systematically classify contributions to $T_{\mu\nu}$ according to their importance (or better yet their power behavior) in the scattering amplitudes, one needs to order the contributions by their twist.

The leading twist part is isolated from $T^{(a)}_{\mu\nu}$ by symmetrization and trace subtraction on the level of local operators. Here only operators bilinear in the quark fields appear and they will ultimately result in a GPD description of the scattering process (to this accuracy). This is technically not very demanding and can be done within a couple of lines of algebra, cf. [23]. Once one starts to include subleading twist contributions it becomes more complicated. Starting at twist-3 one encounters derivatives of twist-2 operators as well as operators with an additional gluon field. They may be related by equations of motion. Let us consider a simple example by taking the operator (in the convention of Eq. (3.20))

$$\bar{q}(z_1)\gamma_\mu q(z_2)\,. \tag{3.21}$$

as well as the Fock-Schwinger gauge (3.15), then one obtains

$$\begin{aligned}\partial_\nu\big(\bar{q}(z_1)\gamma_\mu q(z_2)\big) &= z_1[\bar{D}_\nu\bar{q}](z_1)\gamma_\mu q(z_2) + z_2\bar{q}(z_1)\gamma_\mu[D_\nu q](z_2)\\ &\quad + igz_{12}\int_0^1 du\, z_{21}^u\bar{q}(z_1)x^\rho F_{\rho\nu}(z_{21}^u)\gamma_\mu q(z_2)\,,\end{aligned} \tag{3.22}$$

where

$$\begin{aligned}D_\mu &= \partial_\mu - igA^a_\mu T^a\,,\\ \bar{D}_\mu &= \partial_\mu + igA^a_\mu (T^a)^t\,.\end{aligned} \tag{3.23}$$

Although rather schematic, the above equation (3.22) is an example of the entanglement of a total derivative of a two-particle operator with a higher twist operator containing more fields, as present in $T^{(b)}_{\mu\nu}$. Apart from the explicit appearance of $\bar{q}Fq$-type operators in $T^{(b)}_{\mu\nu}$, they also appear implicitly in $T^{(a)}_{\mu\nu}$, starting from twist-3. The lesson to be learned is that operators of the type $\bar{q}Fq$ do not necessarily give rise to genuine higher twist multi-particle correlations, but contain descendants of the leading twist operators. The latter are relevant for the power corrections in hard exclusive processes and one would like to have them separated from the dynamical higher twist sector. Such a separation is feasible because the dynamical or *quasipartonic* operators obey an autonomous set of renormalization group equations. Putting them to zero at one scale ensures that they do not reappear at another scale.

The separation of the contributions of interest turns out to be surprisingly difficult. In Refs. [23,24] this problem has been addressed and solved. A helpful input that went into this calculation was the renormalization group of twist-4 operators at one-loop accuracy, cf. [34,35]. The operators that are relevant for the power corrections fall into the class of so-called *nonquasipartonic operators*. One of the major results given in [34,35] was the proof that there exists a certain scalar product for the quasipartonic sector, which has the property that the matrix of anomalous dimensions is Hermitian w.r.t. this product. This feature was identified to be a consequence of conformal invariance of massless QCD at one-loop order. Hermiticity implies orthogonality of the anomalous dimension eigenvectors. It was shown that this property is sufficient to separate the relevant contributions, thus bypassing the direct diagonalization of the renormalization kernels, which is probably a very difficult task. Of course, for this approach to work in practice the explicit knowledge of the scalar product is necessary which is also available in [34,35]. The actual derivation is then basically reduced to an algebraic problem and can be found in [23]. We will merely quote the results and take the opportunity to introduce necessary notation.

3.2. Operator product expansion to kinematic twist-4 accuracy

The results of [23] are most conveniently presented in the spinor formalism, see Ch. 2. To this end we write for the time-ordered product

$$T_{\alpha\dot\alpha\beta\dot\beta} = i\,\mathrm{T}\big\{j_{\alpha\dot\alpha}(z_1x)j_{\beta\dot\beta}(z_2x)\big\}\,, \tag{3.24}$$

and its expansion to the order $1/x^2$ can be cast into the form

$$\begin{aligned} T_{\alpha\dot\alpha\beta\dot\beta} = -\frac{2}{\pi^2(z_{12})^3x^4}\Big[&x_{\alpha\dot\beta}\mathbb{B}_{\beta\dot\alpha}(z_1,z_2) - x_{\beta\dot\alpha}\mathbb{B}_{\alpha\dot\beta}(z_2,z_1) + x_{\alpha\dot\beta}x_{\beta\dot\alpha}\big(\mathbb{A}(z_1,z_2)-\mathbb{A}(z_2,z_1)\big) \\ &+ x^2\big(x_{\beta\dot\beta}\partial_{\alpha\dot\alpha}\mathbb{C}(z_1,z_2) - x_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\mathbb{C}(z_2,z_1)\big)\Big]\,. \end{aligned} \tag{3.25}$$

Here the derivative $\partial_{\alpha\dot\alpha} = \partial_\mu(\sigma^\mu)_{\alpha\dot\alpha}$ acts w.r.t. the space-time point x. The operators $\mathbb{A}$, $\mathbb{C}$ are pure twist-4 corrections and $\mathbb{B}$ is a sum of all twists from two to four, which we conveniently label by

$$\mathbb{B}_{\alpha\dot\alpha}(z_1,z_2) = \mathbb{B}^{t=2}_{\alpha\dot\alpha}(z_1,z_2) + \mathbb{B}^{t=3}_{\alpha\dot\alpha}(z_1,z_2) + \mathbb{B}^{t=4}_{\alpha\dot\alpha}(z_1,z_2)\,. \tag{3.26}$$

Explicit expressions for $\mathbb{A}$, $\mathbb{B}$, $\mathbb{C}$ will be given below, after a couple of preparatory definitions.

First we define the vector and axial-vector operators

$$\begin{aligned} \mathcal{O}_V(z_1,z_2) &= \bar q(z_1x)\not{x}q(z_2x)\,, \\ \mathcal{O}_A(z_1,z_2) &= \bar q(z_1x)\not{x}\gamma_5q(z_2x)\,, \end{aligned} \tag{3.27}$$

as well as their (anti-)symmetrized versions

$$\begin{aligned} \mathcal{O}_{V,-}(z_1,z_2) &= \mathcal{O}_V(z_1,z_2) - \mathcal{O}_V(z_2,z_1)\,, \\ \mathcal{O}_{A,+}(z_1,z_2) &= \mathcal{O}_A(z_1,z_2) + \mathcal{O}_A(z_2,z_1)\,. \end{aligned} \tag{3.28}$$

It is necessary to have a projection of $\mathcal{O}_{\dots}$ on leading twist operators. To this end one makes use of a (real) auxiliary light-like vector n, which can always be represented with the help of two spinors $\lambda, \bar\lambda$

$$n^\mu = \frac{1}{2}(\sigma^\mu)_{\alpha\dot\alpha}\lambda^\alpha\bar\lambda^{\dot\alpha}\,,\quad n^2=0\,, \tag{3.29}$$

with $\bar\lambda = \lambda^\dagger$, see Ch. 2. Completely equivalent is of course $n_{\alpha\dot\alpha} = \lambda_\alpha\bar\lambda_{\dot\alpha}$. The particular combination of axial and vector operators that will enter in the OPE is denoted by

$$\mathcal{O}_{++}(z_1,z_2) = \bar\psi_+(z_1n)\psi_+(z_2n) - \chi_+(z_2n)\bar\chi_+(z_1n)\,, \tag{3.30}$$

where

$$\psi_+ = \lambda^\alpha \psi_\alpha\,, \quad \bar\psi_+ = \bar\lambda^{\dot\alpha}\bar\psi_{\dot\alpha}\,,$$
$$\chi_+ = \lambda^\alpha \chi_\alpha\,, \quad \bar\chi_+ = \bar\lambda^{\dot\alpha}\bar\chi_{\dot\alpha}\,. \tag{3.31}$$

Note that $\mathcal{O}_{++}(z_1, z_2)$ itself is twist-2 and it is equivalent to

$$\mathcal{O}_{++}(z_1, z_2) = \frac{1}{2}\big[\mathcal{O}_{V,-}(z_1, z_2) - \mathcal{O}_{A,+}(z_1, z_2)\big]_{x\to n}\,. \tag{3.32}$$

The separation of the leading twist contributions from $\mathcal{O}_{\dots}$ as function of x, which is not light-like, is achieved by the *leading twist projector* Π. Let $\varphi(x)$ be an arbitrary operator and $\varphi(\lambda, \bar\lambda)$ its restriction on the light-ray n, expressed in terms of $\lambda, \bar\lambda$. Then Π is defined as follows:

$$[\Pi\varphi](x) = \Pi(x, \lambda)\varphi(\lambda, \bar\lambda) = \sum_{k=0}^{\infty} \frac{(\bar\partial\bar x\partial)^k}{(k!)^2}\varphi(\lambda, \bar\lambda)\Big|_{\lambda=\bar\lambda=0}\,, \tag{3.33}$$

where

$$(\bar\partial\bar x\partial) = \frac{\partial}{\partial\bar\lambda^{\dot\alpha}}\bar x^{\dot\alpha\alpha}\frac{\partial}{\partial\lambda^\alpha}\,. \tag{3.34}$$

The application of the leading twist projection to any operator will be denoted by the superscript "$t = 2$", e.g.

$$\mathcal{O}^{t=2}_{++}(z_1, z_2) = \Pi(x, \lambda)\mathcal{O}_{++}(z_1, z_2)\,. \tag{3.35}$$

Technically, Π implements the symmetrization and subtraction of traces for the Lorentz indices in each coefficient in the Taylor expansion of $\mathcal{O}_{++}(z_1, z_2)$. Since the nonlocal operator $\mathcal{O}_{++}(z_1, z_2)$ can be interpreted as the generating function for local operators, one may think of $[\Pi\mathcal{O}_{++}](z_1, z_2)$ as the generating function for local twist-2 operators. To see that $[\Pi\mathcal{O}_{++}](z_1, z_2)$ has the desired properties, consider the k-th term in the expansion (3.33). It reads

$$\frac{1}{(k!)^2}\bar x^{\dot\alpha_1\alpha_1}\cdots\bar x^{\dot\alpha_k\alpha_k}\frac{\partial}{\partial\bar\lambda^{\dot\alpha_1}}\cdots\frac{\partial}{\partial\bar\lambda^{\dot\alpha_k}}\frac{\partial}{\partial\lambda^{\alpha_1}}\cdots\frac{\partial}{\partial\lambda^{\alpha_k}}\mathcal{O}_{++}(z_1, z_2)\Big|_{\lambda=\bar\lambda=0}\,. \tag{3.36}$$

The tensor

$$\frac{\partial}{\partial\bar\lambda^{\dot\alpha_1}}\cdots\frac{\partial}{\partial\bar\lambda^{\dot\alpha_k}}\frac{\partial}{\partial\lambda^{\alpha_1}}\cdots\frac{\partial}{\partial\lambda^{\alpha_k}}\mathcal{O}_{++}(z_1, z_2) \tag{3.37}$$

is symmetric under an arbitrary exchange of two dotted or two undotted indices. Therefore it is also symmetric under the simultaneous exchange of two pairs of indices $(\alpha_i, \dot\alpha_i) \leftrightarrow (\alpha_j, \dot\alpha_j)$. Interchanging two such pairs in this way is equivalent to the interchange of two Lorentz indices in the usual vector formalism. Tracelessness is also easy to see, since taking a trace with respect to two Lorentz indices corresponds to the contraction with a dotted and an undotted ε-symbol.

The upcoming calculations can become rather cumbersome, if one insists on using the very definition of Π in Eq. (3.33). In App. A we give a proof of a simpler representation of Π accurate to the order x^2, to which all calculations are done. It shall be stressed here, that this is only for convenience and using Eq. (3.33) directly is also possible. However, as repeatedly pointed out in [23], translation and gauge invariance is only guaranteed to work at twist-4 level, see also Sec. 3.3, thus there is no real need to cope with the "full" expression of Π.

With these preliminaries we can now proceed to give explicit formulas for (3.25) and (3.26).

3.2.1. Twist-2

The twist-2 contribution to the OPE is entirely contained in $\mathbb{B}_{\alpha\dot\alpha}$, see Eq. (3.26), and is given by

$$\mathbb{B}^{t=2}_{\alpha\dot\alpha}(z_1,z_2) = \frac{1}{2}\partial_{\alpha\dot\alpha}\int_0^1 du\, \mathcal{O}^{t=2}_{++}(uz_1,uz_2)\,. \tag{3.38}$$

It originates from the leading twist projection of $T^{(a)}_{\mu\nu}$ in Eq. (3.19).

3.2.2. Twist-3

In this sector we have

$$\begin{aligned}\mathbb{B}^{t=3}_{\alpha\dot\alpha}(z_1,z_2) = \frac{1}{4}\int_0^1 du\, u\int_{z_2}^{z_1}\frac{dv}{z_{12}}\Big\{&z_1\big((x\bar\sigma^\mu\partial)_{\alpha\dot\alpha} + \ln u\,\partial_{\alpha\dot\alpha}x^2\partial^\mu\big)\big[i\mathbf{P}_\mu,\mathcal{O}^{t=2}_{++}(uz_1,uv)\big]\\ &+ z_2\big((\bar x\sigma^\mu\bar\partial)_{\dot\alpha\alpha} + \ln u\,\partial_{\alpha\dot\alpha}x^2\partial^\mu\big)\big[i\mathbf{P}_\mu,\mathcal{O}^{t=2}_{++}(uv,uz_2)\big]\Big\}\,,\end{aligned} \tag{3.39}$$

where $[\mathbf{P}_\mu,\dots]$ stands for the commutator with the momentum operator $\mathbf{P}_\mu$. If one evaluates this operator in the basis of momentum eigenstates, one may simply replace $[\mathbf{P}_\mu,\dots]$ by the difference of eigenmomenta between final and initial state. Note that the contributions $\sim \ln u$ are twist-4 terms, whose role is to subtract twist-4 contaminations from the rest, such that $\mathbb{B}^{t=3}_{\alpha\dot\alpha}$ is purely twist-3.

3.2.3. Twist-4

Let us now give the twist-4 contributions, $\mathbb{A}$, $\mathbb{B}^{t=4}_{\alpha\dot\alpha}$ and $\mathbb{C}$. Out of the multitude of equivalent representations that are given in [23], we will pick the most convenient one for our calculations.

The term $\mathbb{A}(z_1,z_2)$, entering antisymmetrized in $z_1\leftrightarrow z_2$, reads

$$\begin{aligned}\mathbb{A}(z_1,z_2) = \frac{1}{4}\int_0^1 du\,\Big[&z_1z_2u^2\ln u\,\mathcal{O}_1(uz_1,uz_2)\\ &+\Big(z_2\partial_{z_2} - \frac{z_1}{z_{12}} - \ln u\, z_2\partial^2_{z_2}z_{12}\Big)\mathcal{R}(uz_1,uz_2)\\ &-\Big(z_1\partial_{z_1} - \frac{z_2}{z_{21}} - \ln u\, z_1\partial^2_{z_1}z_{21}\Big)\bar{\mathcal{R}}(uz_1,uz_2)\Big]\,,\end{aligned} \tag{3.40}$$

with

$$\mathcal{R}(z_1,z_2) = z_{12}\int_{z_2}^{z_1}\frac{dv}{z_{12}}\int_{z_2}^{v}\frac{dw}{z_{12}}\,\frac{w-z_2}{z_1-w}\left[\frac{1}{2}S_+\mathcal{O}_1(v,w)-(S_0-1)\mathcal{O}_2(v,w)\right],$$
$$\bar{\mathcal{R}}(z_1,z_2) = z_{12}\int_{z_2}^{z_1}\frac{dv}{z_{12}}\int_{z_2}^{v}\frac{dw}{z_{12}}\,\frac{z_1-v}{v-z_2}\,\left[\frac{1}{2}S_+\mathcal{O}_1(v,w)-(S_0-1)\mathcal{O}_2(v,w)\right]. \tag{3.41}$$

Here the operators $\mathcal{O}_{1,2}$ are defined as

$$\mathcal{O}_1(z_1,z_2) = \left[i\mathbf{P}^\mu,\left[i\mathbf{P}_\mu,\mathcal{O}^{t=2}_{++}(z_1,z_2)\right]\right],$$
$$\mathcal{O}_2(z_1,z_2) = \left[i\mathbf{P}^\mu,\partial_\mu\mathcal{O}^{t=2}_{++}(z_1,z_2)\right]. \tag{3.42}$$

and S_+, S_0 are given by

$$S_+ = v^2\partial_v + w^2\partial_w + 2(v+w)\,.$$
$$S_0 = v\partial_v + w\partial_w + 2\,. \tag{3.43}$$

Formally, S_+, S_0 are generators of conformal transformations on the light-ray, acting on products of fields with light-cone positions v, w and conformal spin $j=1$, i.e. here $S_+ \equiv S_+^{(1,1)}$, $S_0 \equiv S_0^{(1,1)}$ with the general form

$$S_+^{(j_1,j_2)} = v^2\partial_v + w^2\partial_w + 2(j_1 v + j_2 w)\,,$$
$$S_0^{(j_1,j_2)} = v\partial_v + w\partial_w + j_1 + j_2\,. \tag{3.44}$$

The next contribution is

$$\mathbb{B}^{t=4}_{\alpha\dot\alpha}(z_1,z_2) = x^2\partial_{\alpha\dot\alpha}\mathbb{B}^{t=4}(z_1,z_2)\,, \tag{3.45}$$

where

$$\begin{aligned}\mathbb{B}^{t=4}(z_1,z_2) = \frac{1}{8}\int_0^1\frac{du}{u^2}\Bigg\{& u^2(1-u^2+u^2\ln u)z_1z_2\mathcal{O}_1(uz_1,uz_2)\\ &-\left[(1-u^2)\left(z_2\partial_{z_2}-\frac{z_1}{z_{12}}\right)+(1-u^2+u^2\ln u)z_2\partial^2_{z_2}z_{12}\right]\mathcal{R}(uz_1,uz_2)\\ &+\left[(1-u^2)\left(z_1\partial_{z_1}-\frac{z_2}{z_{21}}\right)+(1-u^2+u^2\ln u)z_1\partial^2_{z_1}z_{21}\right]\bar{\mathcal{R}}(uz_1,uz_2)\Bigg\}.\end{aligned} \tag{3.46}$$

Finally the last term reads

$$\mathbb{C}(z_1,z_2) = -\frac{1}{8}\int_0^1\frac{du}{u^2}\left(\mathcal{R}(uz_1,uz_2)+\bar{\mathcal{R}}(uz_2,uz_1)\right). \tag{3.47}$$

3.3. On gauge invariance and translations

The finite-Q^2 corrections to the OPE in the case of deep inelastic scattering have first been calculated in [25] and are occasionally called *Nachtmann corrections*. Technically their origin lies in the "subtraction of traces"-prescription of the leading twist projection. Since DIS can be described by the forward Compton amplitude $\langle p|\,T_{\mu\nu}\,|p\rangle$, the situation is somewhat simpler than for off-forward reactions. Forward matrix elements of operators, which are total derivatives, vanish because they are always proportional to the momentum difference between the two proton states. Therefore for DIS the only nonzero contribution stems from $\mathbb{B}^{t=2}_{\alpha\dot\alpha}$ in Eq. (3.38). For the off-forward reactions like DVCS the corrections due to the trace subtraction terms alone are not sufficient, they are in fact unphysical. To see this, we first establish general relations for the time-ordered product of currents.

The tensor $T_{\mu\nu}(z_1, z_2)$ in Eq. (3.1) (here we make the dependence on $z_{1,2}$ explicit) inherits its translational properties from the fundamental fields $\bar q(x), q(x)$, which gives

$$T_{\mu\nu}(z_1 + \delta z, z_2 + \delta z) = e^{i\delta z(\mathbf{P}x)} T_{\mu\nu}(z_1, z_2) e^{-i\delta z(\mathbf{P}x)} \,. \tag{3.48}$$

The infinitesimal version of Eq. (3.48) is

$$(\partial_{z_1} + \partial_{z_2}) T_{\mu\nu}(z_1, z_2) = \big[i(\mathbf{P}x), T_{\mu\nu}(z_1, z_2)\big] \,. \tag{3.49}$$

Later, we will give an equivalent relation for (3.48), (3.49) in momentum space. In that framework it can be seen explicitly, that for the Nachtmann-type corrections Eqs. (3.48), (3.49) do not hold. The full translational invariance is only restored when one takes into account all other corrections of twist-3 and twist-4. Eqs. (3.48), (3.49) are then valid up to corrections of twist-5 or higher.

Similarly the (electromagnetic) gauge invariance, which implies current conservation,

$$\partial^\mu j_\mu(x) = 0 \,, \tag{3.50}$$

requires

$$\begin{aligned}
\partial^\mu T_{\mu\nu}(z_1, z_2) &= \mathrm{T}\big\{j_\mu(z_1 x)\partial^\mu j_\nu(z_2 x)\big\} = z_2\big[i\mathbf{P}^\mu, T_{\mu\nu}(z_1, z_2)\big] \,, \\
\partial^\nu T_{\mu\nu}(z_1, z_2) &= \mathrm{T}\big\{j_\nu(z_2 x)\partial^\nu j_\mu(z_1 x)\big\} = z_1\big[i\mathbf{P}^\nu, T_{\mu\nu}(z_1, z_2)\big] \,.
\end{aligned} \tag{3.51}$$

Here the same subtlety as before arises: The Ward identities (3.51) are fulfilled only in the sum of all twists. This property is known since quite some time and was first noticed in Refs. [36–39] independently.

It can be checked that Eqs. (3.49), (3.51) hold up to corrections of twist-5 or higher. The condition of gauge invariance will be used from the very beginning when we define the helicity dependent scattering amplitudes. On the other hand the translation property (3.48) will not be exploited at any time. The final result should reflect (3.48) automatically, which provides a very strong check of the calculation, as we shall see below.

4. Deeply virtual Compton scattering

4.1. Basics

Our target application of the OPE formulated in the previous chapter is the deeply virtual Compton scattering. It is defined as the scattering of a virtual photon on a hadron with a real photon in the final state,

$$h(p) + \gamma^*(q) \to h(p') + \gamma(q')\,. \tag{4.1}$$

Here q (q') and p (p') denote the initial (final) momenta of the participating particles. Their respective polarizations are left implicit. DVCS appears as a subprocess of the exclusive lepton-hadron scattering, see Fig. 4.1. The biggest share of all available experiments on DVCS were performed using protons, and it seems that they will also be the prime target of interest in the foreseeable future. Thus we consider a nucleon target for definiteness, although everything in this work will be valid for any spin-$\frac{1}{2}$ baryon. We will comment on the case of scalar targets, e.g. pions, in Sec. 6.3.

At leading order the theoretical description of DVCS is formulated in terms of the hadronic *Compton tensor*

$$\mathcal{A}_{\mu\nu} = i \int d^4x\, e^{-iqx}\, \langle p'|\,\mathrm{T}\{j_\mu(x) j_\nu(0)\}\,|p\rangle\ . \tag{4.2}$$

On diagrammatic level the photon fields will couple to the Lorentz indices μ, ν. The scattering occurs between the photons and a quark or antiquark being emitted from the proton and reabsorbed in the final state. This is occasionally called the "handbag mechanism", see Fig. 4.1. Applying the OPE from Ch. 3 to the r.h.s. of (4.2) gives the amplitude tensor in terms of short distance coefficients and nonperturbative input from the proton. This was one of the main tasks in this work, and we will continue to present the essential steps towards the answer in the kinematic twist-4 approximation.

To make progress, we introduce a some conventional notation, starting with kinematical variables. The final state photon is assumed to be real, i.e.

$$(q')^2 = 0\,. \tag{4.3}$$

It turns out that instead of working directly with the individual hadronic momenta p, p' it is convenient to introduce the somewhat standard vectors P and Δ, being the average and difference of p, p' respectively,

$$\begin{aligned} P_\mu &= \frac{1}{2}(p_\mu + p'_\mu)\,, \\ \Delta_\mu &= p'_\mu - p_\mu = q_\mu - q'_\mu\,, \end{aligned} \tag{4.4}$$

The hard scale of the process is the virtuality of the initial state photon,

$$Q^2 = -q^2\,, \quad Q^2 > 0\,. \tag{4.5}$$

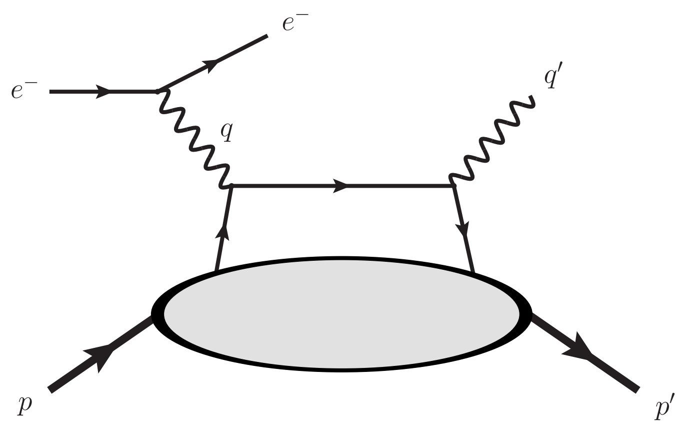

Figure 4.1.: Schematic diagram ("handbag mechanism") for the DVCS amplitude $h(p) + \gamma^*(q) \to h(p') + \gamma(q')$ of a hadron h. The virtual photon is emitted from an electron beam.

The invariant square of the momentum transfer is denoted by

$$t = \Delta^2 \,, \quad t \leq 0 \,. \tag{4.6}$$

In any hard scattering process it is convenient to classify two light-like vectors $n, \bar{n}$ and directions transverse to the thusly chosen light-cone. The vectors n and $\bar{n}$ are used to define large "plus" and small "minus" components for fast moving particles. We choose them in such a way, that they can be expressed in terms of the photon momenta only. Since we have a real photon in the final state, it seems natural to take

$$n_\mu = q'_\mu \,, \quad n^2 = 0 \,, \tag{4.7}$$

while second light-like vector is constructed as a linear combination of the two photon momenta

$$\bar{n}_\mu = (1 - \epsilon) q'_\mu - q_\mu \,, \quad \bar{n}^2 = 0 \,, \tag{4.8}$$

where

$$\epsilon = \frac{t}{Q^2 + t} \,. \tag{4.9}$$

In the Bjorken limit, ϵ can be regarded as a small number. The normalization of n and $\bar{n}$ is given by

$$(n\bar{n}) = \frac{1}{2}(Q^2 + t) \,, \tag{4.10}$$

and thus is of the order of the hard scale. As a consequence, the momentum difference Δ has only longitudinal components,

$$\Delta_\mu = -\bar{n}_\mu - \epsilon n_\mu \,. \tag{4.11}$$

The only independent vector that has transverse components is P, which we write as

$$P_\mu = \frac{1}{2\xi}(\bar{n}_\mu - \epsilon n_\mu) + P_{\perp,\mu}\,, \tag{4.12}$$

where the so-called *skewness* variable ξ is defined as

$$\xi = -\frac{(\Delta q')}{2(Pq')} = \frac{(pn)-(p'n)}{(pn)+(p'n)}\,. \tag{4.13}$$

From the positivity of (physical) "plus" momenta, in particular $(pn) \geq 0$ and $(p'n) \geq 0$, one immediately deduces the support $\xi \in [-1,1]$. For DVCS we can even make a stronger restriction: let x_B be the *Bjorken scaling variable* defined by

$$x_B = \frac{Q^2}{2(pq)} = \frac{Q^2}{2(p'q')+Q^2}\,, \quad x_B \in [0,1]\,. \tag{4.14}$$

Using momentum conservation one finds

$$\xi = \frac{x_B\big(1+\frac{t}{Q^2}\big)}{2-x_B\big(1-\frac{t}{Q^2}\big)}\,, \tag{4.15}$$

i.e. in the Bjorken limit when $Q^2 \gg |t|$ the support of ξ is restricted to $\xi \in [0,1]$. The square of the transverse component of P is given by

$$P_\perp^2 = -\vec{P}_\perp^2 = m^2 + \frac{t}{4}\frac{1-\xi^2}{\xi^2}\,, \tag{4.16}$$

where m is the mass of the hadron. The inequality $P_\perp^2 < 0$ induces an inequality on t, m, ξ,

$$|t| \geq |t_{\min}| \equiv \frac{4m^2\xi^2}{1-\xi^2}\,. \tag{4.17}$$

Let us now transfer these kinematics to the spinor formalism of Ch. 2. The first observation is that a light-like vector can be represented as a "pure" product of spinors $\sim u_\alpha v_{\dot\alpha}$ and, since n and $\bar{n}$ are real-valued vectors, one can choose two auxiliary spinors λ_α and μ_α to write

$$\begin{aligned} n_\mu \sigma^\mu_{\alpha\dot\alpha} &= \lambda_\alpha \bar\lambda_{\dot\alpha}\,, \\ \bar{n}_\mu \sigma^\mu_{\alpha\dot\alpha} &= \mu_\alpha \bar\mu_{\dot\alpha}\,, \end{aligned} \tag{4.18}$$

where $\bar\lambda_{\dot\alpha} = \lambda^\dagger_\alpha$ and $\bar\mu_{\dot\alpha} = \mu^\dagger_\alpha$. Note that we re-use the symbol λ here, which is not to be confused with the one from Sec. 3.2. The normalization of λ, μ reads

$$(\lambda\mu)(\bar\mu\bar\lambda) = 2(n\bar{n})\,. \tag{4.19}$$

Any transverse, i.e. orthogonal to the light-cone, degree of freedom can also be expressed with the help of these spinors. A possible choice of the basis in the transverse plane is

$$\lambda_\alpha \bar\mu_{\dot\alpha}\,, \quad \mu_\alpha \bar\lambda_{\dot\alpha}\,, \tag{4.20}$$

such that given an arbitrary four vector x_μ, its associated matrix $x_{\alpha\dot\alpha} = x_\mu \sigma^\mu_{\alpha\dot\alpha}$ can be expanded in the following way

$$(\lambda\mu)(\bar\mu\bar\lambda)x_{\alpha\dot\alpha} = x_{++}\mu_\alpha\bar\mu_{\dot\alpha} + x_{--}\lambda_\alpha\bar\lambda_{\dot\alpha} - x_{-+}\lambda_\alpha\bar\mu_{\dot\alpha} - x_{+-}\mu_\alpha\bar\lambda_{\dot\alpha}\,, \tag{4.21}$$

with

$$\begin{aligned} x_{++} &= (\lambda x\bar\lambda)\,, \quad x_{--} = (\mu x\bar\mu)\,, \\ x_{-+} &= (\mu x\bar\lambda)\,, \quad x_{+-} = (\lambda x\bar\mu)\,, \end{aligned} \tag{4.22}$$

following the conventions of Eq. (2.10), e.g. $(\lambda x\bar\lambda) = \lambda^\alpha x_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}$ and so on. Note that the transverse basis in the spinor formalism can be related to the following construction in the usual vector formalism: Out of n_μ and $\bar n_\mu$ can define two projectors on the transverse plane,

$$g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{\bar n_\mu n_\nu + n_\mu\bar n_\nu}{(n\bar n)}\,, \quad \varepsilon^\perp_{\mu\nu} = \frac{1}{(n\bar n)}\varepsilon_{\mu\nu\rho\sigma}\bar n^\rho n^\sigma\,. \tag{4.23}$$

They satisfy the relations

$$g^{\perp\nu}_\mu g^\perp_{\nu\rho} = g^\perp_{\mu\rho}\,, \quad \varepsilon^{\perp\nu}_\mu\varepsilon^\perp_{\nu\rho} = -g^\perp_{\mu\rho}\,. \tag{4.24}$$

We will use a shorthand notation for the action of these projectors, viz.

$$\begin{aligned} x^\perp_\mu &= g^\perp_{\mu\nu}x^\nu\,, \\ \tilde x^\perp_\mu &= \varepsilon^\perp_{\mu\nu}x^\nu \end{aligned} \tag{4.25}$$

for any x. In the spinor notation one finds

$$\begin{aligned} g^\perp_{\alpha\dot\alpha\beta\dot\beta} &\equiv (\sigma^\mu)_{\alpha\dot\alpha}(\sigma^\nu)_{\beta\dot\beta}g^\perp_{\mu\nu} = \frac{-2}{(\lambda\mu)(\bar\mu\bar\lambda)}(\lambda_\alpha\bar\lambda_{\dot\alpha}\mu_\beta\bar\mu_{\dot\beta} + \mu_\alpha\bar\mu_{\dot\alpha}\lambda_\beta\bar\lambda_{\dot\beta}) - 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\,, \\ \varepsilon^\perp_{\alpha\dot\alpha\beta\dot\beta} &\equiv (\sigma^\mu)_{\alpha\dot\alpha}(\sigma^\nu)_{\beta\dot\beta}\varepsilon^\perp_{\mu\nu} = \frac{2i}{(\lambda\mu)(\bar\mu\bar\lambda)}(\mu_\alpha\bar\lambda_{\dot\alpha}\lambda_\beta\bar\mu_{\dot\beta} - \lambda_\alpha\bar\mu_{\dot\alpha}\mu_\beta\bar\lambda_{\dot\beta})\,, \end{aligned} \tag{4.26}$$

and according to Eq. (4.21)

$$\begin{aligned} (\lambda\mu)(\bar\mu\bar\lambda)x^\perp_{\alpha\dot\alpha} &= -(x_{-+}\lambda_\alpha\bar\mu_{\dot\alpha} + x_{+-}\mu_\alpha\bar\lambda_{\dot\alpha})\,, \\ (\lambda\mu)(\bar\mu\bar\lambda)\tilde x^\perp_{\alpha\dot\alpha} &= -i(x_{-+}\lambda_\alpha\bar\mu_{\dot\alpha} - x_{+-}\mu_\alpha\bar\lambda_{\dot\alpha})\,. \end{aligned} \tag{4.27}$$

Thus we can also write an equivalent representation of the transverse projector $g^\perp$,

$$g^\perp_{\alpha\dot\alpha\beta\dot\beta} = \frac{-2}{(\lambda\mu)(\bar\mu\bar\lambda)}(\mu_\alpha\bar\lambda_{\dot\alpha}\lambda_\beta\bar\mu_{\dot\beta} + \lambda_\alpha\bar\mu_{\dot\alpha}\mu_\beta\bar\lambda_{\dot\beta})\,. \tag{4.28}$$

4.2. Helicity amplitudes

Given the time-ordered product of electromagnetic currents at positions x and y we define its off-forward nucleon matrix element by

$$M_{\mu\nu}(x,y) \equiv \langle p', s'|\, i\,\mathrm{T}\{j_\mu(x)j_\nu(y)\}\,|p,s\rangle\,. \tag{4.29}$$

The matrix element $M_{\mu\nu}(x,y)$ transforms under translations as

$$e^{-i(p'-p)z} M_{\mu\nu}(x,y) = M_{\mu\nu}(x-z, y-z)\,, \tag{4.30}$$

which essentially means that the dependence of $M_{\mu\nu}(x,y)$ on one position variable is always trivial. By considering the Fourier transform in both positions, the aforementioned property manifests itself in a momentum-conserving δ-function

$$\int d^4x \int d^4y\, e^{-iqx+iq'y} M_{\mu\nu}(x,y) = (2\pi)^4 \delta(q+p-q'-p') \int d^4x\, e^{-i(z_1 q - z_2 q')x} T_{\mu\nu}(z_1,z_2)\,, \tag{4.31}$$

where $T_{\mu\nu}(z_1,z_2)$ is given by Eq. (3.1) and

$$z_{12} \equiv z_1 - z_2 = 1\,. \tag{4.32}$$

The expression on the r.h.s. of Eq. (4.31) suggests the definition of the amplitude tensor $\mathcal{A}_{\mu\nu}$:

$$\mathcal{A}_{\mu\nu} = \int d^4x\, e^{-i(z_1 q - z_2 q')x}\, T_{\mu\nu}(z_1,z_2)\,. \tag{4.33}$$

In the following we always imply the momentum conservation on $\mathcal{A}_{\mu\nu}$, although we did not include the δ-function of Eq. (4.31) in the definition (4.33). Under this condition and by using (3.48) and (4.32) it is easy to be seen that $\mathcal{A}_{\mu\nu}$ does not depend on z_1 (or z_2). As a consequence, $\mathcal{A}_{\mu\nu}$ coincides with the preliminary definition (4.2). It further enables us to choose the value of one of the variables, say z_1, completely arbitrary and adjust the other one according to (4.32). Popular choices in the literature are $z_1 = 1, z_2 = 0$ or $z_1 = 0, z_2 = -1$ or $z_1 = -z_2 = \frac{1}{2}$. However we shall go without this simplification. Instead z_1 and z_2 are kept under the condition $z_{12} = 1$, and the independence on z_i is reserved as very powerful check of the result. The strength of this check was emphasized in Sec. 3.3. Generally, with a few exceptions, we expect a cancellation of the z_i-dependencies in the sum of all twists, i.e. at the very end of a typically long calculation.

To make further progress we expand $\mathcal{A}_{\mu\nu}$ in a suitably chosen basis. The starting point is the observation that gauge invariance implies for the amplitude in momentum space

$$\begin{aligned}
(z_1 q^\mu - z_2 q'^\mu)\mathcal{A}_{\mu\nu} &= z_2(q^\mu - q'^\mu)\mathcal{A}_{\mu\nu}\,,\\
(z_1 q^\nu - z_2 q'^\nu)\mathcal{A}_{\mu\nu} &= z_1(q^\nu - q'^\nu)\mathcal{A}_{\mu\nu}\,,
\end{aligned} \tag{4.34}$$

and therefore obviously

$$\begin{aligned}
q^\mu \mathcal{A}_{\mu\nu} &= 0\,,\\
q'^\nu \mathcal{A}_{\mu\nu} &= 0\,.
\end{aligned} \tag{4.35}$$

To arrive at Eq. (4.34) one can represent momentum vector as a derivative w.r.t. x acting on the exponential. Integration by parts, where the boundary terms vanish, and Eq. (3.51) verify Eq. (4.34).

To formulate these requirements in the spinor formalism, recall that the second equation of (4.35) means that

$$\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta} \equiv \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta}\mathcal{A}_{\mu\nu} \tag{4.36}$$

needs to have at least one factor of λ_β or $\bar\lambda_{\dot\beta}$,

$$\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta} = \lambda_\beta \bar\mu_{\dot\beta}\mathcal{A}^{(1)}_{\alpha\dot\alpha} + \mu_\beta\bar\lambda_{\dot\beta}\mathcal{A}^{(2)}_{\alpha\dot\alpha} + \lambda_\beta\bar\lambda_{\dot\beta}\mathcal{A}^{(3)}_{\alpha\dot\alpha}\,. \tag{4.37}$$

The first constraint of (4.35) reads

$$\big((1-\epsilon)\lambda^\alpha\bar\lambda^{\dot\alpha} - \mu^\alpha\bar\mu^{\dot\alpha}\big)\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta} = 0 \tag{4.38}$$

and puts a limitation on the available structures $\mathcal{A}^{(i)}_{\alpha\dot\alpha}$ $(i = 1, 2, 3)$. They can be written as follows:

$$\mathcal{A}^{(i)}_{\alpha\dot\alpha} = \lambda_\alpha\bar\mu_{\dot\alpha}\mathcal{A}^{(i)}_1 + \mu_\alpha\bar\lambda_{\dot\alpha}\mathcal{A}^{(i)}_2 + \big((1-\epsilon)\lambda_\alpha\bar\lambda_{\dot\alpha} + \mu_\alpha\bar\mu_{\dot\alpha}\big)\mathcal{A}^{(i)}_3\,. \tag{4.39}$$

Therefore we have found nine possible structures, which can be isolated by suitable contractions with the auxiliary spinors. We define the helicity conserving amplitudes (the name will become clear below)

$$\begin{aligned}
\mathcal{A}_{++} &= \frac{1}{4(n\bar n)}\lambda^\alpha\bar\mu^{\dot\alpha}\mu^\beta\bar\lambda^{\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,\\
\mathcal{A}_{--} &= \frac{1}{4(n\bar n)}\mu^\alpha\bar\lambda^{\dot\alpha}\lambda^\beta\bar\mu^{\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,
\end{aligned} \tag{4.40}$$

and the helicity flip amplitudes

$$\begin{aligned}
\mathcal{A}_{+-} &= \frac{1}{4(n\bar n)}\lambda^\alpha\bar\mu^{\dot\alpha}\lambda^\beta\bar\mu^{\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,\\
\mathcal{A}_{-+} &= \frac{1}{4(n\bar n)}\mu^\alpha\bar\lambda^{\dot\alpha}\mu^\beta\bar\lambda^{\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,
\end{aligned} \tag{4.41}$$

as well as the longitudinal-to-transverse helicity flip amplitudes

$$\begin{aligned}
\mathcal{A}_{0+} &= -\frac{\big((1-\epsilon)\lambda^\alpha\bar\lambda^{\dot\alpha} + \mu^\alpha\bar\mu^{\dot\alpha}\big)\mu^\beta\bar\lambda^{\dot\beta}}{4\sqrt{2(1-\epsilon)}(n\bar n)}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,\\
\mathcal{A}_{0-} &= -\frac{\big((1-\epsilon)\lambda^\alpha\bar\lambda^{\dot\alpha} + \mu^\alpha\bar\mu^{\dot\alpha}\big)\lambda^\beta\bar\mu^{\dot\beta}}{4\sqrt{2(1-\epsilon)}(n\bar n)}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,.
\end{aligned} \tag{4.42}$$

There are three more amplitudes

$$\begin{aligned}
\mathcal{A}_{+0} &= -\frac{\mu^\alpha\bar\lambda^{\dot\alpha}\mu^\beta\bar\mu^{\dot\beta}}{4\sqrt{2(1-\epsilon)Q^2}(n\bar n)}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,\\
\mathcal{A}_{-0} &= -\frac{\lambda^\alpha\bar\mu^{\dot\alpha}\mu^\beta\bar\mu^{\dot\beta}}{4\sqrt{2(1-\epsilon)Q^2}(n\bar n)}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,\\
\mathcal{A}_{00} &= +\frac{\mu^\alpha\bar\mu^{\dot\alpha}\mu^\beta\bar\mu^{\dot\beta}}{8(1-\epsilon)\sqrt{Q^2}(n\bar n)}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,,
\end{aligned} \tag{4.43}$$

which do not have any physical significance, as we shall see. The relations between $\mathcal{A}_{ij}$ $(i, j \in \{0, \pm\})$ and $\mathcal{A}^{(l)}_k$ $(k, l \in \{1, 2, 3\})$ are straightforward, e.g. $\mathcal{A}_{++} = (n\bar n)A^{(1)}_2$, etc. For the most part of this work the results will be presented in terms of $\mathcal{A}_{ij}$.

Let us recall the meaning of the transverse basis in Eq. (4.20): the vectors corresponding to $\lambda_\alpha\bar\mu_{\dot\alpha}$ and $\mu_\alpha\bar\lambda_{\dot\alpha}$ are related by complex conjugation and are orthogonal to n and $\bar n$ and thus orthogonal to q and q'. Then the normalized vectors associated with

$$\begin{aligned}\varepsilon^+_{\alpha\dot\alpha} &= \frac{\mu_\alpha\bar\lambda_{\dot\alpha}}{\sqrt{(n\bar n)}}\,,\\ \varepsilon^-_{\alpha\dot\alpha} &= \frac{\lambda_\alpha\bar\mu_{\dot\alpha}}{\sqrt{(n\bar n)}}\end{aligned} \tag{4.44}$$

can be identified with the two physical photon helicities. In addition the third option

$$\varepsilon^0_{\alpha\dot\alpha} = \frac{(1-\epsilon)\lambda^\alpha\bar\lambda^{\dot\alpha} + \mu^\alpha\bar\mu^{\dot\alpha}}{\sqrt{2(1-\epsilon)(n\bar n)}} \tag{4.45}$$

corresponds to a longitudinal polarization.

In the vector notation the decomposition of $\mathcal{A}_{\mu\nu}$ reads

$$\begin{aligned}\mathcal{A}_{\mu\nu} = {}& \varepsilon^+_\mu\varepsilon^-_\nu\mathcal{A}_{++} + \varepsilon^-_\mu\varepsilon^+_\nu\mathcal{A}_{--} + \varepsilon^0_\mu\varepsilon^-_\nu\mathcal{A}_{0+} + \varepsilon^0_\mu\varepsilon^+_\nu\mathcal{A}_{0-} + \varepsilon^+_\mu\varepsilon^+_\nu\mathcal{A}_{+-} + \varepsilon^-_\mu\varepsilon^-_\nu\mathcal{A}_{-+}\\ &+ \varepsilon^+_\mu q'_\nu\mathcal{A}_{+0} + \varepsilon^-_\mu q'_\nu\mathcal{A}_{-0} + \varepsilon^0_\mu q'_\nu\mathcal{A}_{00}\end{aligned} \tag{4.46}$$

where $\varepsilon^-_\mu = \left(\varepsilon^+_\mu\right)^*$ and

$$\left(\varepsilon^\pm\right)^2 = \left(\varepsilon^\pm\varepsilon^0\right) = 0\,, \quad \left(\varepsilon^0\right)^2 = -\left(\varepsilon^+\varepsilon^-\right) = 1\,. \tag{4.47}$$

The amplitudes $\mathcal{A}_{\pm\pm}$ describe the process of scattering a photon with helicity $\pm$ off a nucleon where the final state photon helicity is the same ($\pm$). $\mathcal{A}_{0\pm}$ characterize the transition from a longitudinally polarized photon (this degree of freedom is allowed for virtual photons) into a final state photon with helicity $\pm$. Finally the amplitudes $\mathcal{A}_{+0}$, $\mathcal{A}_{-0}$, $\mathcal{A}_{00}$, though nonzero in general, do not contribute to any DVCS observable, since they correspond to a longitudinally polarized photon the in the final state.

Note that $\mathcal{A}_{\mu\nu}$ is dimensionless and we expect in the limit $Q\to\infty$ the following schematic power counting behavior:

$$\mathcal{A}_{\pm\pm} \sim \mathcal{O}(Q^0)\,, \quad \mathcal{A}_{0\pm} \sim \mathcal{O}(m/Q, \sqrt{t}/Q)\,, \quad \mathcal{A}_{\pm\mp} \sim \mathcal{O}(m^2/Q^2, t/Q^2)\,. \tag{4.48}$$

Formally there are helicity flip contributions at NLO through the gluon transversity distribution, which contribute at $\mathcal{O}(Q^0)$ to $\mathcal{A}_{\pm\mp}$. They are irrelevant for our purposes and can be ignored. If the calculation is done in the OPE to twist-4 accuracy in the sense of the kinematic approximation, one can obtain the "leading" contribution to the helicity flip amplitudes and the "leading plus subleading" contribution to $\mathcal{A}_{\pm\pm} \sim \mathcal{O}(Q^0)+\mathcal{O}(m^2/Q^2, t/Q^2)$. Working out these corrections will be one of the main new results, while there exist expressions for $\mathcal{A}_{0\pm}$ and $\mathcal{A}_{\pm\mp}$ in the literature, cf. [37,40]. We try to compare results in much detail in Sec. 6.4.

One can express the polarization vectors in terms of the momenta of the DVCS process. This can be easily written down for the longitudinal vector ε^0,

$$\varepsilon^0_\mu = -\frac{1}{Q}q_\mu - \frac{Q}{(qq')}q'_\mu\,, \tag{4.49}$$

which, by construction, depends only on the photon vectors. To obtain a representation in the transverse plane, one can use the average proton momentum P. By using Eqs. (4.44) and (4.27) one finds

$$\varepsilon^{\pm}_{\mu} = -\frac{1}{2(P\varepsilon^{\mp})}(P^{\perp}_{\mu} \pm i\bar{P}^{\perp}_{\mu})\,, \tag{4.50}$$

where $P^{\perp}_{\mu}$ and $\bar{P}^{\perp}_{\mu}$ are obtained with the help of the projectors $g^{\perp}_{\mu\nu}$ and $\varepsilon^{\perp}_{\mu\nu}$. The latter read in terms of the momenta

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q'_\nu + q'_\mu q_\nu}{(qq')} + \frac{q'_\mu q'_\nu\, q^2}{(qq')^2}\,, \quad \varepsilon^{\perp}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}\frac{q^\rho q'^\sigma}{(qq')}\,. \tag{4.51}$$

Since $P_\perp$ can be expanded as

$$P_\perp = -(\varepsilon^+ P)\varepsilon^- - (\varepsilon^- P)\varepsilon^+\,, \tag{4.52}$$

one finds

$$2(\varepsilon^+ P)(\varepsilon^- P) = |P_\perp|^2\,. \tag{4.53}$$

Note that we still have some freedom in the choice of basis, namely a rotation in the transverse plane. In the spinor notation, one can redefine

$$\begin{aligned} \lambda &\to e^{i\phi_1}\lambda\,, \quad \bar{\lambda} \to e^{-i\phi_1}\bar{\lambda}\,, \\ \mu &\to e^{i\phi_2}\mu\,, \quad \bar{\mu} \to e^{-i\phi_2}\bar{\mu}\,, \end{aligned} \tag{4.54}$$

with arbitrary phases ϕ_1, ϕ_2. This leaves the light-cone invariant, but maps

$$\begin{aligned} \varepsilon^+ &\to e^{i(\phi_2-\phi_1)}\varepsilon^+\,, \\ \varepsilon^- &\to e^{i(\phi_1-\phi_2)}\varepsilon^-\,. \end{aligned} \tag{4.55}$$

The amplitude tensor $\mathcal{A}_{\mu\nu}$ does not change under such redefinitions, as we shall see explicitly. In more detail, we expect $\mathcal{A}_{0\pm} \sim (\varepsilon_\pm P)$ and $\mathcal{A}_{\mp\pm} \sim (\varepsilon_\pm P)^2$. We can use this freedom to make the transverse components of $P_\perp$ degenerate, i.e.

$$(\varepsilon^+ P) = (\varepsilon^- P) = -\frac{|P_\perp|}{\sqrt{2}} \tag{4.56}$$

and thus

$$\varepsilon^{\pm}_{\mu} = \frac{P^{\perp}_{\mu} \pm i\bar{P}^{\perp}_{\mu}}{\sqrt{2}|P_\perp|}\,. \tag{4.57}$$

4.3. Generalized parton distributions in a nutshell

Throughout the entire work we will stick to the convention of M. Diehl, see [15]. The four nucleon GPDs, H^q, E^q, $\widetilde{H}^q$, $\widetilde{E}^q$, for a given quark flavor q are defined as matrix elements of bilinear quark field operators "living" on the light-ray n, expressed in momentum space via a Fourier transform:

$$
\begin{aligned}
\mathfrak{F}^q &= \int \frac{dz}{4\pi}\, e^{-ixz(P\mathfrak{n})} \, \langle p'|\, \bar{q}\Big(\frac{1}{2}z\mathfrak{n}\Big) \not{\mathfrak{n}} q\Big(-\frac{1}{2}z\mathfrak{n}\Big)\, |p\rangle \\
&= \frac{1}{2(P\mathfrak{n})}\bigg[H^q(x,\xi,t)\bar{u}(p')\not{\mathfrak{n}}u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{\mu\nu}\mathfrak{n}_\mu\Delta_\nu}{2m}u(p)\bigg], \\
\widetilde{\mathfrak{F}}^q &= \int \frac{dz}{4\pi} e^{-ixz(P\mathfrak{n})} \, \langle p'|\, \bar{q}\Big(\frac{1}{2}z\mathfrak{n}\Big) \not{\mathfrak{n}}\gamma_5 q\Big(-\frac{1}{2}z\mathfrak{n}\Big)\, |p\rangle \\
&= \frac{1}{2(P\mathfrak{n})}\bigg[\widetilde{H}^q(x,\xi,t)\bar{u}(p')\not{\mathfrak{n}}\gamma_5 u(p) + \widetilde{E}^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5(\Delta\mathfrak{n})}{2m}u(p)\bigg],
\end{aligned}
\tag{4.58}
$$

where the spin quantum numbers are suppressed in the notation. Here the light-like vector $\mathfrak{n}$ can be taken proportional to $\mathfrak{n} \propto n$. In fact Eq. (4.58) is invariant under an arbitrary re-parametrization $\mathfrak{n} \to \alpha\mathfrak{n}$. A GPD depends on the renormalization scale μ^2 (always left implicit) and on the three kinematical variables (x, ξ, t). Note that here x refers to a momentum fraction variable, not to be confused with the position variable introduced earlier. This unfortunate clash of notation is rather standard in the literature and will hopefully not result in a confusion. It should be clear from the context to which quantity "x" refers to.

For convenience the following abbreviations for the spinor bilinears are introduced:

$$
\begin{aligned}
v_\mu &= \bar{u}(p')\gamma_\mu u(p)\,, & s &= \bar{u}(p')u(p)\,, \\
\tilde{v}_\mu &= \bar{u}(p')\gamma_\mu\gamma_5 u(p)\,, & \tilde{s} &= \bar{u}(p')\gamma_5 u(p)\,,
\end{aligned}
\tag{4.59}
$$

and by using the Dirac equation one can perform a rewriting of the definitions in Eq. (4.58)

$$
\begin{aligned}
\mathfrak{F}^q &= \frac{1}{2(P\mathfrak{n})}\bigg[(v\mathfrak{n})(H^q + E^q) - \frac{(P\mathfrak{n})s}{m}\,E^q\bigg], \\
\widetilde{\mathfrak{F}}^q &= \frac{1}{2(P\mathfrak{n})}\bigg[(\tilde{v}\mathfrak{n})\widetilde{H}^q + \frac{(\Delta\mathfrak{n})\tilde{s}}{2m}\,\widetilde{E}^q\bigg],
\end{aligned}
\tag{4.60}
$$

in this context also known as *Gordon decomposition*.

In order to readily apply the OPE formalism from Ch. 3 we employ the *double distribution representation* of the hadronic matrix elements [12,13,41]. The notation closely follows that of [16] (slightly adapted to our needs),

$$
\begin{aligned}
&\langle p'|\, \bar{q}(z_1\mathfrak{n})\not{\mathfrak{n}}q(z_2\mathfrak{n})\,|p\rangle = \\
&\quad = \int d\beta\, d\alpha\, e^{-i\ell_{12}\mathfrak{n}}\bigg[(v\mathfrak{n})h^q(\beta,\alpha) + \frac{is}{z_{12}m}\Big(\partial_\beta f^q(\beta,\alpha) + \partial_\alpha g^q(\beta,\alpha)\Big)\bigg], \\
&\langle p'|\, \bar{q}(z_1\mathfrak{n})\not{\mathfrak{n}}\gamma_5 q(z_2\mathfrak{n})\,|p\rangle = \\
&\quad = \int d\beta\, d\alpha\, e^{-i\ell_{12}\mathfrak{n}}\bigg[(\tilde{v}\mathfrak{n})\tilde{h}^q(\beta,\alpha) + \frac{i\tilde{s}}{z_{12}m}\Big(\partial_\beta \tilde{f}^q(\beta,\alpha) + \partial_\alpha \tilde{g}^q(\beta,\alpha)\Big)\bigg],
\end{aligned}
\tag{4.61}
$$

where

$$
\ell_{12} = \beta z_{21}P + \frac{1}{2}\big(z_{12}\alpha - z_1 - z_2\big)\Delta\,. \tag{4.62}
$$

The region of integration in Eq. (4.61) goes over the square $|\alpha| + |\beta| \le 1$, see Fig. 4.2. The functions h^q, $\tilde{h}^q$, f^q, $\tilde{f}^q$, g^q and $\tilde{g}^q$ in Eq. (4.61) will be referred to as *double distributions* (DDs). For convenience it will be assumed that they vanish at the border of the integration

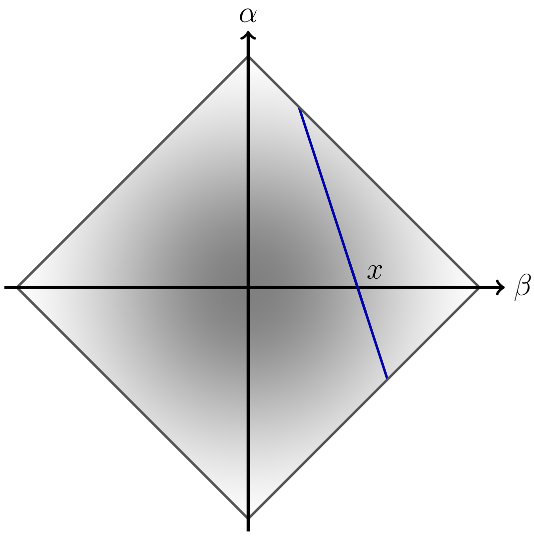

Figure 4.2.: Square shaped support region for double distributions in the (β, α)-plane. Integration along the blue line with slope $-1/\xi$ converts the DD into a GPD at point (x, ξ).

$|\alpha| + |\beta| = 1$. This assumption is not strictly necessary, it was made to trade Lorentz invariant prefactors $(P n)$ and (Δn) (analogously to Eq. (4.60)) for derivatives w.r.t. β and α acting on the exponential. Integration by parts with zero boundary terms yields (4.61). This step will be "undone" in the final expressions, so that possible boundary terms (if existing) would cancel again. Alternatively, without changing the final results, one could have kept the factors $(P n)$ and (Δn) from the beginning.

A connection between the DDs and GPDs themselves can be obtained by comparing Eq. (4.61) with Eq. (4.58). One example of such a relation is

$$\int d\beta\, d\alpha\, \delta(x - \beta - \alpha\xi) \tilde{h}^q(\beta, \alpha) = \widetilde{H}^q(x, \xi, t)\,. \tag{4.63}$$

In other words, to obtain the GPD $\widetilde{H}$ at (x, ξ), one needs to do a line integral, parametrized by $x-\beta-\alpha\xi = 0$ in the (β, α)-plane, see Fig. 4.2. We discuss more details and generalizations in App. C.

Using Hermiticity properties, time reversal and parity symmetry, one can show[1] for the vector operator

$$\begin{aligned}
&\langle p', s'|\, \bar{q}(z_1 n) \not{n} q(z_2 n)\, |p, s\rangle^* = \\
&\quad = \langle p, s|\, \bar{q}(z_2 n) \not{n} q(z_1 n)\, |p', s'\rangle \\
&\quad = \langle p', -s'|\, \bar{q}(-z_1 n) \not{n} q(-z_2 n)\, |p, -s\rangle\, (-1)^{\delta_{s,-s'}}\,,
\end{aligned} \tag{4.64}$$

where s, s' are the spins of the nucleon states (previously suppressed in the notation). An

[1]See Ref. [16] for a proof.

analogous equation holds for the axial operator:

$$\begin{aligned}\langle p', s'|\, \bar{q}(z_1 n)\not{n}\gamma_5 q(z_2 n)\, |p, s\rangle^* = \\ = \langle p, s|\, \bar{q}(z_2 n)\not{n}\gamma_5 q(z_1 n)\, |p', s'\rangle = \\ = \langle p', -s'|\, \bar{q}(-z_1 n)\not{n}\gamma_5 q(-z_2 n)\, |p, -s\rangle\, (-1)^{\delta_{s,s'}} \,. \end{aligned} \tag{4.65}$$

As a consequence, the GPDs are real and even in ξ

$$\big[F^q(x, \xi, t)\big]^* = F^q(x, -\xi, t) = F^q(x, \xi, t)\,, \quad F^q \in \{H^q, E^q, \widetilde{H}^q, \widetilde{E}^q\}\,. \tag{4.66}$$

Along the same lines one deduces, that the double distributions are real and obey the symmetry relations

$$\begin{aligned}\big[h^q(\beta, \alpha)\big]^* &= h^q(\beta, -\alpha) = h^q(\beta, \alpha)\,, \\ \big[\tilde{h}^q(\beta, \alpha)\big]^* &= \tilde{h}^q(\beta, -\alpha) = \tilde{h}^q(\beta, \alpha)\,, \\ \big[\Phi^q(\beta, \alpha)\big]^* &= \Phi^q(\beta, -\alpha) = \Phi^q(\beta, \alpha)\,, \\ \big[\tilde{\Phi}^q(\beta, \alpha)\big]^* &= -\tilde{\Phi}^q(\beta, -\alpha) = \tilde{\Phi}^q(\beta, \alpha)\,, \end{aligned} \tag{4.67}$$

where

$$\begin{aligned}\Phi^q(\beta, \alpha) &= \partial_\beta f^q(\beta, \alpha) + \partial_\alpha g^q(\beta, \alpha)\,, \\ \tilde{\Phi}^q(\beta, \alpha) &= \partial_\beta \tilde{f}^q(\beta, \alpha) + \partial_\alpha \tilde{g}^q(\beta, \alpha)\,. \end{aligned} \tag{4.68}$$

As a consequence of Lorentz invariance GPDs have to satisfy *polynomiality conditions*, meaning that the n-th Mellin moment of H^q and E^q has to be a polynomial in ξ of degree n,

$$\begin{aligned}\int_{-1}^{1} dx\, x^{n-1} H^q(x, \xi, t) &= \sum_{i=0}^{n-1} c_i^{H^q}(t)\xi^i + c_n^q(t)\xi^n\,, \\ \int_{-1}^{1} dx\, x^{n-1} E^q(x, \xi, t) &= \sum_{i=0}^{n-1} c_i^{E^q}(t)\xi^i - c_n^q(t)\xi^n\,. \end{aligned} \tag{4.69}$$

Here the coefficients with odd indices have to vanish due to the symmetry (4.66). Note that the same coefficient in front of the highest power of ξ enters with opposite sign for the Mellin moment for H^q and E^q. There exist analogous relations to Eq. (4.69) for $\widetilde{H}^q$ and $\widetilde{E}^q$, with the modification that the ξ^n term is absent. It is important to realize that Eq. (4.69) is an extremely strong constraint on the functional dependence of the GPDs on the momentum fractions x and ξ. Via Eqs. (C.1), (C.2) one can see that the polynomiality condition is automatically satisfied for a DD ansatz. This is a big advantage when one constructs models for GPDs. The peculiar form in Eq. (4.68) accounts for the existence of a pion-pole contribution for $\widetilde{E}^q$ and a D-term for H^q and E^q. The latter also allows for a nonzero coefficient of the highest power ξ^n in Eq. (4.69). A DD ansatz in Eq. (4.61) is also very convenient for our purposes, although for a rather pragmatic reason, namely that the dependence on the light-cone vector n is encoded in a rather simple way, making the application of the leading twist projector straightforward.

The GPDs H^q and $\widetilde{H}^q$ reduce to the usual unpolarized and polarized parton densities in

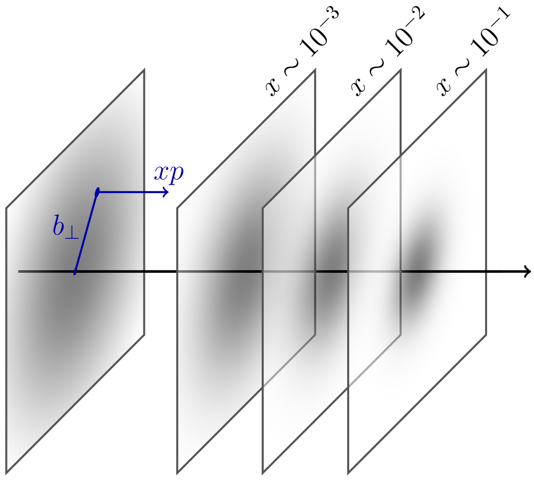

Figure 4.3.: Schematic cartoon of the impact parameter representation $q(x, b_\perp)$. For given x the distribution $q(x, b_\perp)$ gives the probability density to find a quark q inside a fast moving nucleon, separated by a distance $b_\perp$ from the center of motion and sharing a fraction x of the nucleon momentum p.

the forward limit, e.g. for $x > 0$

$$
\begin{aligned}
H^q(x, \xi = 0, t = 0) &= q(x)\,, \\
\widetilde{H}^q(x, \xi = 0, t = 0) &= \Delta q(x)\,.
\end{aligned}
\tag{4.70}
$$

Since $q(x)$ and $\Delta q(x)$ are well-determined by DIS experiments, this reduction formula gives a strong constraint on H^q and $\widetilde{H}^q$. On the other hand, for E^q and $\widetilde{E}^q$ there is no such constraint from DIS. Furthermore GPDs fully encode the Dirac and Pauli form factors F_1^q and F_2^q (for a particular quark flavor q) by means of their first Mellin moments,

$$
\begin{aligned}
\int_{-1}^{1} dx\, H^q(x, \xi, t) &= F_1^q(t)\,, \\
\int_{-1}^{1} dx\, E^q(x, \xi, t) &= F_2^q(t)\,.
\end{aligned}
\tag{4.71}
$$

The ξ-independence of this relation follows readily from the polynomiality conditions (4.69) with vanishing coefficients c_1^q. Analogous relations exist between $\widetilde{H}$ ($\widetilde{E}$) and the axial (pseudoscalar) form factors g_A^q (g_P^q).

As it was noticed by M. Burkardt in [19,20], going over to so-called impact parameter representation $q(x, b_\perp)$,

$$
q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} H^q(x, \xi = 0, -\vec{\Delta}_\perp^2)
\tag{4.72}
$$

one gets a probabilistic distribution of a quark q with momentum fraction x located at a distance $b_\perp = \left|\vec{b}_\perp\right|$ from the center of transverse momentum, see Fig. 4.3. The description is valid in the infinite momentum frame and here we have considered the special case $\xi = 0$, though it can be generalized to nonvanishing skewness, see [21] for details. In order to get such a “tomographic” nucleon image in the spirit of Fig. 4.3, on needs to know the t-dependence of the GPDs in a sufficient range, see Eq. (4.72). This calls for an experimental program to explore the large $|t|$ regime of the phase space. As we have stressed already, one should take into account the corrections $\sim t/Q^2$, which requires a twist-4 calculation. We proceed with this task in the next chapter.

5. Calculation of helicity amplitudes

5.1. Notation

In terms of double distributions the $z_{1,2}$-dependence of the nucleon matrix elements like $\langle p', s' | \, \bar{q}(z_1 n) \not{n} q(z_2 n) \, | p, s \rangle$ is encoded in a somewhat convenient way. Recall that the vector/axial operator always enters the OPE anti-/symmetrized in z_1, z_2. Exchanging z_1 with z_2 effectively amounts to a replacement $(\beta, \alpha) \to (-\beta, -\alpha)$ of the DDs. Using the symmetry relations (4.67), we are in practice always working with the following DDs

$$\begin{aligned}
h_-^q(\beta, \alpha) &= \frac{1}{2}\big(h^q(\beta, \alpha) - h^q(-\beta, \alpha)\big) \, , \\
\tilde{h}_+^q(\beta, \alpha) &= \frac{1}{2}\big(\tilde{h}^q(\beta, \alpha) + \tilde{h}^q(-\beta, \alpha)\big) \, , \\
\Phi_+^q(\beta, \alpha) &= \frac{1}{2}\big(\Phi^q(\beta, \alpha) + \Phi^q(-\beta, \alpha)\big) \, , \\
\tilde{\Phi}_-^q(\beta, \alpha) &= \frac{1}{2}\big(\tilde{\Phi}^q(\beta, \alpha) + \tilde{\Phi}^q(-\beta, \alpha)\big) \, .
\end{aligned} \tag{5.1}$$

The subscripts "$+,-$" denote the parity under simultaneous reflection in β and α, e.g. $h_-^q(-\beta, -\alpha) = -h_-^q(\beta, \alpha)$. For convenience we will also introduce an abbreviated notation for the DD integral,

$$\begin{aligned}
\langle p' | \, \mathcal{O}_{++}(z_1, z_2) \, | p \rangle = (vn) \int dh_- \, e^{-i\ell_{12} n} + \frac{is}{z_{12} m} \int d\Phi_+ \, e^{-i\ell_{12} n} \\
-(\tilde{v} n) \int d\tilde{h}_+ \, e^{-i\ell_{12} n} - \frac{i\tilde{s}}{z_{12} m} \int d\tilde{\Phi}_- \, e^{-i\ell_{12} n} \, .
\end{aligned} \tag{5.2}$$

Here, the quantities h_-, $\tilde{h}_-$, Φ_+, $\tilde{\Phi}_-$ and $\mathcal{O}_{++}$ are understood to be charge (squared) averaged, i.e.

$$h_- = \sum_q e_q^2 \, h_-^q \, , \text{ etc.} \tag{5.3}$$

with e_q being the fractional electric charge of quark flavor q, e.g. $e_u = \frac{2}{3}$. The measure dh_- is defined by

$$dh_- = d\beta \, d\alpha \, h_-(\beta, \alpha) \tag{5.4}$$

and analogously for the three other DDs.

One more notational shortcut will be employed, which saves us from a couple of tedious, not very enlightening definitions. We will use the same symbols for operators and their matrix elements, i.e. from now on we identify e.g.

$$\mathbb{B}(z_1, z_2) \to \langle p' | \, \mathbb{B}(z_1, z_2) \, | p \rangle \, , \text{ etc.,} \tag{5.5}$$

which will hopefully not result in a confusion.

5.2. Transverse helicity flip $\mathcal{A}_{\pm\mp}$

To begin we consider the amplitudes describing the photon helicity flip transition, cf. Eq. (4.41)

$$\mathcal{A}_{\pm\mp} = \frac{1}{4(n\bar{n})}\mathcal{A}_{\pm\mp\pm\mp}\,, \quad \mathcal{A}_{+-+-} = \lambda^\alpha\bar{\mu}^{\dot\alpha}\lambda^\beta\bar{\mu}^{\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,, \quad \mathcal{A}_{-+-+} = \mu^\alpha\bar{\lambda}^{\dot\alpha}\mu^\beta\bar{\lambda}^{\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,. \tag{5.6}$$

Given that $\mathcal{A}_{\pm\mp}$ is "helicity nonconserving", it is already a suppressed contribution compared to the helicity conserving ones. To our stated accuracy it is sufficient to consider only the contributions from $\mathbb{B}$ (twist-2 and twist-3), which can be summarized as

$$\mathcal{A}_{\pm\mp\pm\mp} = -2\int \frac{d^4x\, e^{-irx}}{\pi^2(x^2-i0)^2}x_{\pm\mp}\left[\mathbb{B}_{\pm\mp}(z_1,z_2) - \mathbb{B}_{\pm\mp}(z_2,z_1)\right], \tag{5.7}$$

where

$$r = z_1 q - z_2 q' = z_{12}q + z_2\Delta\,. \tag{5.8}$$

At this point we remind that $\mathbb{B}$ is now understood to be "sandwiched" between proton momentum eigenstates. Corrections from $\mathbb{A}$, $\mathbb{B}^{t=4}$ and $\mathbb{C}$ would give rise to terms of order $\mathcal{O}(Q^{-3})$, which are beyond our approximation and can be disregarded here. For the same reason it suffices to approximate the leading twist projector by unity, cf. Eq. (A.11), $\Pi \to \mathbb{1}$.

In order to keep the calculations manageable, we consider each contribution of a given DD separately. Note that in Eq. (5.7) $\mathbb{B}$ appears antisymmetrized in $z_{1,2}$. On the twist-2 level, this antisymmetrization is directly passed on to the operator (or its matrix element) and thus there is no $t=2$ contribution from the axial sector.

5.2.1. Twist-2

Using the $z_1 \leftrightarrow z_2$ symmetry and Eq. (3.38), one arrives at the following expression

$$\mathcal{A}^{t=2}_{\pm\mp\pm\mp} = -2\int \frac{d^4x\, e^{-irx}}{\pi^2(x^2-i0)^2}x_{\pm\mp}\partial_{\pm\mp}\int_0^1 du\, \mathcal{O}_{V,-}(uz_1,uz_2)\,. \tag{5.9}$$

At first we look only at the h_- contribution, which reads in our compactified notation:

$$\mathcal{A}^{t=2}_{\pm\mp\pm\mp}[h_-] = -2\int_0^1 du \int dh_- \int \frac{d^4x\, e^{-i(r+u\ell_{12})x}}{\pi^2(x^2-i0)^2}x_{\pm\mp}\big(v_{\pm\mp} - iu\ell_{12,\pm\mp}(vx)\big)\,, \tag{5.10}$$

where we changed the order of integrations and carried out the derivative $\partial_{\pm\mp}$ explicitly, using

$$\partial_{\alpha\dot\alpha}x_{\beta\dot\beta} = -2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\,. \tag{5.11}$$

To indicate a certain contribution of a DD to an amplitude we have used the functional brackets, "$[h_-]$" in the above example, and we will adopt this notation in what follows. The Fourier integral can now be evaluated with the help of Eqs. (B.2) and (B.3), yielding after a little algebra

$$\mathcal{A}^{t=2}_{\pm\mp\pm\mp}[h_-] = -8\int_0^1 du \int dh_- \left[\frac{u\beta P_{\pm\mp} v_{\pm\mp}}{(r+u\ell_{12})^2 + i0} + \frac{(u\beta P_{\pm\mp})^2 (r+u\ell_{12}, v)}{((r+u\ell_{12})^2+i0)^2}\right]. \tag{5.12}$$

Note that the only transverse component of ℓ_{12} is through $\beta z_{21} P$, see (4.62). We silently put $z_{21} = -1$ according to the discussions in Secs. 3.3 and 4.2.

The intermediate result (5.12) still needs to be expanded in Q. We take the opportunity to give an extended discussion on how to do this quickly for the case at hand and for all upcoming amplitudes. By straightforward algebra one obtains for the squares (z_1 and z_2 arbitrary) and their asymptotic scaling behavior

$$\begin{aligned} r^2 &= -2z_1(n\bar{n}) + z_1^2 t = \mathcal{O}(Q^2)\,, \\ \ell_{12}^2 &= -|P_\perp|^2\beta^2 z_{12}^2 + t(z_{12}\omega - z_1)(z_{12}(\alpha-\omega) - z_2) = \mathcal{O}(t, m^2)\,. \end{aligned} \tag{5.13}$$

Here ω is defined as

$$\omega = \frac{1}{2}\left(\frac{\beta}{\xi} + \alpha + 1\right). \tag{5.14}$$

The cross term is

$$(r\ell_{12}) = -(n\bar{n})(z_{12}\omega - z_1) + tz_1\big(z_{12}(\omega - \beta/(2\xi)) - z_1\big) \tag{5.15}$$

and therefore also of order $\mathcal{O}(Q^2)$. In addition, we also need to give a hierarchy of the spinor bilinears. This can be done by temporarily assuming that the light-cone vector n defines the large plus momenta for particles moving almost with the speed of light along the z-axis. Using explicit representations for $u_\lambda(p)$, cf. Ch. 2, one gets for the scalar

$$\begin{aligned} \bar{u}_\uparrow(p')u_\uparrow(p) = \bar{u}_\downarrow(p')u_\downarrow(p) &= m\sqrt{\frac{(pn)}{(p'n)}} + m\sqrt{\frac{(p'n)}{(pn)}}\,, \\ \bar{u}_\uparrow(p')u_\downarrow(p) = -\big(\bar{u}_\downarrow(p')u_\uparrow(p)\big)^* &= \bar{p}'\sqrt{\frac{(pn)}{(p'n)}} - \bar{p}\sqrt{\frac{(p'n)}{(pn)}} \end{aligned} \tag{5.16}$$

and for the pseudoscalar structure

$$\begin{aligned} \bar{u}_\uparrow(p')\gamma_5 u_\uparrow(p) = \bar{u}_\downarrow(p)\gamma_5 u_\downarrow(p') &= m\sqrt{\frac{(pn)}{(p'n)}} - m\sqrt{\frac{(p'n)}{(pn)}}\,, \\ \bar{u}_\uparrow(p')\gamma_5 u_\downarrow(p) = \big(\bar{u}_\downarrow(p')\gamma_5 u_\uparrow(p)\big)^* &= \bar{p}\sqrt{\frac{(p'n)}{(pn)}} - \bar{p}'\sqrt{\frac{(pn)}{(p'n)}}\,, \end{aligned} \tag{5.17}$$

where in the above equations $\bar{p} = p^1 - ip^2$ and $\bar{p}' = p'^1 - ip'^2$. For the axial- and vector-type bilinears

$$\bar{u}_{\lambda'}(p')\not{n}u_\lambda(p) = (-1)^{\delta_{\lambda,\downarrow}}\bar{u}_{\lambda'}(p')\not{n}\gamma_5 u_\lambda(p) = \frac{Q^2+t}{2}\frac{\sqrt{1-\xi^2}}{\xi}\delta_{\lambda\lambda'}\,. \tag{5.18}$$

We can therefore conclude that roughly speaking $(vn), (\tilde{v}n) \sim \mathcal{O}(Q^2)$ and $s, \tilde{s} \sim \mathcal{O}(|P_\perp|)$.

Further, using the Dirac equation, one finds

$$(P^{\perp}v) = ms + \frac{\epsilon}{\xi}(vn)\,,$$
$$(\bar{P}^{\perp}v) = -i(m\tilde{s} + \epsilon(\tilde{v}n))\,. \tag{5.19}$$

From the above discussion it is easy to see, that both terms in Eq. (5.12) are of the same order. Also $(q + u\ell_{12}, v) \approx z_{12}(vn)$ and the following approximation is valid:

$$(r + u\ell_{12})^2 \approx -2(n\bar{n})(z_{12}u\omega + z_1\bar{u})\,. \tag{5.20}$$

Then one arrives at the compact result

$$\mathcal{A}^{t=2}_{\pm\mp\pm\mp}[h_-] = 2\int dh_-\left[\frac{2\beta P_{\pm\mp}v_{\pm\mp}}{(n\bar{n})} + \frac{(\beta P_{\pm\mp})^2(vn)}{(n\bar{n})^2}\partial_\omega\right]\partial_\omega U(\omega)\,, \tag{5.21}$$

where

$$U(\omega) = \int_0^1 du\ln(u\omega + \bar{u}z_1 - i0) = \frac{\omega\ln(\omega - i0) - z_1\ln z_1}{\omega - z_1}\,. \tag{5.22}$$

Apparently, $\mathcal{A}^{t=2}_{\pm\mp\pm\mp}[h_-]$ still depends on z_1. This dependence is canceled by adding the twist-3 contribution, which is calculated in the next section. Note that it is important to keep track of the "$i0$ prescription". Typically one can skip it in intermediate expressions and restore it by the replacement $\omega \to \omega - i0$ in the final results. The "$i0$" in Eq. (5.22) is only necessary inside the logarithm, defining how to approach the logarithmic cut for negative ω. Otherwise $U(\omega)$ is regular.

The contribution from Φ_+ obtained in the same way

$$\mathcal{A}^{t=2}_{\pm\mp\pm\mp}[\Phi_+] = 2\int d\Phi_+\,\frac{(\beta P_{\pm\mp})^2}{(n\bar{n})}\partial_\omega U(\omega)\,. \tag{5.23}$$

5.2.2. Twist-3

We can omit the suppressed twist-4 terms $\sim \ln u$ in Eq. (3.39) and thus the relevant polarization contraction for $\mathbb{B}$ is given by

$$\mathbb{B}^{t=3}_{\pm\mp}(z_1, z_2) = \frac{i}{4}\int_0^1 du\,u\int_{z_2}^{z_1}\frac{dv}{z_{12}}\left[z_1(x\bar{\Delta}\partial)_{\pm\mp}\mathcal{O}^{t=2}_{++}(uz_1, uv) + z_2(\bar{x}\Delta\bar{\partial})_{\mp\pm}\mathcal{O}^{t=2}_{++}(uv, uz_2)\right]. \tag{5.24}$$

By explicit calculation one finds

$$(x\bar{\Delta}\partial)_{\pm\mp} = -(x_{\pm-}\partial_{-\mp} + \epsilon x_{\pm+}\partial_{+\mp})\,,$$
$$(\bar{x}\Delta\bar{\partial})_{\pm\mp} = -(x_{-\pm}\partial_{\mp-} + \epsilon x_{+\pm}\partial_{\mp+})\,. \tag{5.25}$$

Here, all terms proportional to ϵ can be neglected. Also ∂_{--} produces only power-suppressed contributions of the form $(\bar{n}v)$, $(\bar{n}\tilde{v})$, $(\bar{n}\ell_{12})$, $(\bar{n}P)$, $(\bar{n}\Delta)$. Thus we can approximate the square brackets in Eq. (5.24) by $-x_{--}\partial_{+-}z_2\mathcal{O}^{t=2}_{++}(uv, uz_2)$ in the case of $\mathbb{B}^{t=3}_{+-}$ and by $-x_{--}\partial_{-+}z_1\mathcal{O}^{t=2}_{++}(uz_1, uv)$ in the case of $\mathbb{B}^{t=3}_{-+}$. One can re-summarize the resulting expression as

$$\mathbb{B}^{t=3}_{\pm\mp}(z_1,z_2) = -\frac{i}{4}\int_0^1 du\, u \int_{z_2}^{z_1}\frac{dv}{z_{12}}\, x_{--}\partial_{\pm\mp}\Big[z_1\mathcal{O}_V(uz_1,uv) + z_2\mathcal{O}_V(uv,uz_2) \pm z_1\mathcal{O}_A(uz_1,uv) \mp z_2\mathcal{O}_A(uv,uz_2)\Big]\,. \tag{5.26}$$

It is sufficient to continue calculating with the term $\sim z_1\mathcal{O}_V(uz_1,uv)$. The other parts are completely analogous and can be obtained from the former by simple considerations. For definiteness let us start with the contribution from h_-,

$$\mathcal{A}^{t=3}_{\pm\mp\pm\mp}[h_-] = \frac{i}{2}\int dh_- \int_0^1 du\, u \int_{z_2}^{z_1}\frac{dv}{z_{12}} \times \\ \times \int \frac{d^4x\, e^{-irx}}{\pi^2 x^4} x_{\pm\mp}\, x_{--}\partial_{\pm\mp}(vx)\Big(z_1 e^{-iu\ell_{1v}x} + z_2 e^{-iu\ell_{v2}x}\Big), \tag{5.27}$$

where $\ell_{1v} = \ell(z_1, z_2 = v)$ and $\ell_{v2} = \ell(z_1 = v, z_2)$. After carrying out the derivative and the Fourier integral by employing Eqs. (B.3) and (B.4) one obtains after a couple of lines of calculation

$$\mathcal{A}^{t=3}_{\pm\mp\pm\mp}[h_-] = -\int dh_-\left[\frac{2v_{\pm\mp}\beta P_{\pm\mp}}{(n\bar{n})} + \frac{(vn)(\beta P_{\pm\mp})^2}{(n\bar{n})^2}\partial_\omega\right]\partial_\omega V_+(\omega)\,. \tag{5.28}$$

The same approximations as discussed in the twist-2 section were used to arrive at this formula. $V_+(\omega)$ is given by

$$V_\pm(\omega) = \int_0^1 du\, u\int_{z_2}^{z_1}\frac{dv}{z_{12}}\left[\frac{z_2}{z_1 - uv + u(v-z_2)\omega} \pm \frac{z_1}{z_1 - uz_1 + u(z_1 - v)\omega}\right]. \tag{5.29}$$

The contribution of $\tilde{h}_+$ is then easily obtained by taking into account the sign factors $\pm$ and the different symmetrization as well as the simple substitutions $h_- \to \tilde{h}_+$, $v \to \tilde{v}$

$$\mathcal{A}^{t=3}_{\pm\mp\pm\mp}[\tilde{h}_+] = \pm\int d\tilde{h}_+\left[\frac{2\tilde{v}_{\pm\mp}\beta P_{\pm\mp}}{(n\bar{n})} + \frac{(\tilde{v}n)(\beta P_{\pm\mp})^2}{(n\bar{n})^2}\partial_\omega\right]\partial_\omega V_-(\omega)\,, \tag{5.30}$$

with V_- defined above.

Consider now the contribution from Φ_+:

$$\mathcal{A}^{t=3}_{\pm\mp\pm\mp}[\Phi_+] = -\frac{s}{2m}\int d\Phi_+ \int_0^1 du \int_{z_2}^{z_1}\frac{dv}{z_{12}} \times \\ \times \int \frac{d^4x\, e^{-iqx}}{\pi^2 x^4} x_{\pm\mp}x_{--}\partial_{\pm\mp}\left(\frac{z_1}{z_1 - v}e^{-iu\ell_{1v}x} + \frac{z_2}{v - z_2}e^{-iu\ell_{v2}x}\right). \tag{5.31}$$

The basic steps are identical to the ones for h_-, yielding

$$\mathcal{A}^{t=3}_{\pm\mp\pm\mp}[\Phi_+] = -\frac{s}{m}\int d\Phi_+\frac{(\beta P_{\pm\mp})^2}{(n\bar{n})}\partial_\omega V_+(\omega)\,. \tag{5.32}$$

Similar as in the case of $\tilde{h}_+$ the corresponding answer in the $\tilde{\Phi}_-$ sector is given by substituting

$\Phi_+ \to \pm\tilde{\Phi}_-$ and $s \to \tilde{s}$, i.e.

$$\mathcal{A}^{t=3}_{\pm\mp\pm\mp}[\tilde{\Phi}_-] = \mp\frac{\tilde{s}}{m}\int d\Phi_- \frac{(\beta P_{\pm\mp})^2}{(n\bar{n})}\partial_\omega V_-(\omega)\,. \tag{5.33}$$

We finally note that the evaluation of $V_\pm(\omega)$ is elementary and gives

$$\begin{aligned} V_+(\omega) &= \frac{((1-\omega)z_1 - \omega z_2)\ln(\omega - i0)}{(1-\omega)(\omega - z_1)} - \frac{2z_1\ln z_1}{w - z_1}\,, \\ V_-(\omega) &= \frac{\ln(\omega - i0)}{1-\omega}\,, \end{aligned} \tag{5.34}$$

where we silently restored the $i0$ prescription.

5.2.3. Summary

The final expressions for the axial sector involve $V_-(\omega)$ only, which does not depend on z_i as expected from the discussion in Sec. 3.3. For the vector part this symmetry is restored in the sum of twist-2 and twist-3 contributions. This is indeed the case, since

$$2U(\omega) - V_+(\omega) = -\frac{(2\omega - 1)\ln(\omega - i0)}{1-\omega}\,. \tag{5.35}$$

Here one can see the strength of the translation-symmetry check: the z_i-dependence drops out almost in the very last step of the calculation.

Summarizing the result, one can write

$$\begin{aligned} \mathcal{A}_{\pm\mp} = &-\frac{s(P_{\pm\mp})^2}{4m(n\bar{n})^2}\int d\Phi_+\,\beta^2\partial_\omega\frac{(2\omega-1)\ln(\omega - i0)}{\omega - 1} \pm \frac{\tilde{s}(P_{\pm\mp})^2}{4m(n\bar{n})^2}\int d\tilde{\Phi}_-\,\beta^2\partial_\omega\frac{\ln(\omega - i0)}{\omega - 1} \\ &+\frac{P_{\pm\mp}}{4(n\bar{n})^2}\int dh_-\left[2v_{\pm\mp}\beta + \frac{(vn)P_{\pm\mp}\beta^2}{(n\bar{n})}\partial_\omega\right]\partial_\omega\frac{(2\omega-1)\ln(\omega - i0)}{\omega - 1} \\ &\mp\frac{P_{\pm\mp}}{4(n\bar{n})^2}\int d\tilde{h}_+\left[2\tilde{v}_{\pm\mp}\beta + \frac{(\tilde{v}n)P_{\pm\mp}\beta^2}{(n\bar{n})}\partial_\omega\right]\partial_\omega\frac{\ln(\omega - i0)}{\omega - 1}\,. \end{aligned} \tag{5.36}$$

5.3. Longitudinal-to-transverse helicity flip $\mathcal{A}_{0\pm}$

Let us continue with the longitudinal-to-transverse photon helicity transition amplitude $\mathcal{A}_{0\pm}$, from Eq. (4.42). By using (4.38) and $1 - \epsilon \approx 1$, we have to the $1/Q^2$ accuracy

$$\mathcal{A}_{0\pm} \approx -\frac{1}{2\sqrt{2}(n\bar{n})}\mathcal{A}_{--\mp\pm}\,. \tag{5.37}$$

Again, just like in the previous case of $\mathcal{A}_{\pm\mp}$, the leading twist projector is taken as unity, since the corrections of order x^2 to Π would only produce terms that are beyond the twist-4 calculation.

5.3.1. Twist-2

An immediate simplification is achieved by the observation that one can neglect terms proportional to $x_{\pm\mp}\mathbb{B}_{--}$, since $\mathbb{B}_{--}$ will produce terms of order $1/Q^2$ and the presence of transverse components $x_{\pm\mp}$ will ultimately produce a suppression of $P_{\pm\mp}/Q$. Combining these two effects is already beyond the aimed accuracy. Thus we can equate

$$\begin{aligned}\mathcal{A}^{t=2,3}_{---+} &= +2\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4} x_{--}\mathbb{B}^{t=2,3}_{-+}(z_2,z_1)\,,\\ \mathcal{A}^{t=2,3}_{--+-} &= -2\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4} x_{--}\mathbb{B}^{t=2,3}_{+-}(z_1,z_2)\,.\end{aligned} \tag{5.38}$$

The above relations hold for twist-2 and twist-3. From the vector/axial symmetry properties under $z_1 \leftrightarrow z_2$ we obtain

$$\begin{aligned}\mathcal{A}^{t=2}_{--\mp\pm}[h_-] &= -\int dh_- \int_0^1 du \int \frac{d^4x\, e^{-i(r+u\ell_{12})x}}{\pi^2 x^4}\, x_{--}\big(v_{\mp\pm} - iu\ell_{12,\mp\pm}(vx)\big)\,,\\ \mathcal{A}^{t=2}_{--\mp\pm}[\tilde h_+] &= \mp\int d\tilde h_+ \int_0^1 du \int \frac{d^4x\, e^{-i(r+u\ell_{12})x}}{\pi^2 x^4}\, x_{--}\big(\tilde v_{\mp\pm} - iu\ell_{12,\mp\pm}(\tilde vx)\big)\,,\\ \mathcal{A}^{t=2}_{--\mp\pm}[\Phi_+] &= -\frac{s}{m}\int d\Phi_+ \int_0^1 du \int \frac{d^4x\, e^{-i(r+u\ell_{12})x}}{\pi^2 x^4}\, x_{--}\ell_{12,\mp\pm}\,,\\ \mathcal{A}^{t=2}_{--\mp\pm}[\tilde\Phi_-] &= \mp\frac{\tilde s}{m}\int d\tilde\Phi_- \int_0^1 du \int \frac{d^4x\, e^{-i(r+u\ell_{12})x}}{\pi^2 x^4}\, x_{--}\ell_{12,\mp\pm}\,.\end{aligned} \tag{5.39}$$

It should be clear by now how to proceed. The first contribution of (5.39) is evaluated with the help of (B.2) and (B.3) and reads

$$\mathcal{A}^{t=2}_{--\mp\pm}[h_-] = 4(n\bar n)\int dh_- \int_0^1 du \left[\frac{v_{\mp\pm}}{(r+u\ell_{12})^2} + \frac{2(vn)u\beta P_{\mp\pm}}{(r+u\ell_{12})^4}\right]. \tag{5.40}$$

After the usual expansion in $1/Q$ this results in

$$\mathcal{A}^{t=2}_{--\mp\pm}[h_-] = -2\int dh_- \left[v_{\mp\pm} + \frac{(vn)\beta P_{\mp\pm}}{(n\bar n)}\partial_\omega\right] W(\omega)\,, \tag{5.41}$$

where

$$W(\omega) = \int_0^1 \frac{du}{(u\omega + \bar u z_1 - i0)} = \frac{\ln(\omega/z_1 - i0)}{\omega - z_1}\,. \tag{5.42}$$

Obviously, it is possible to write down the answers for all other contributions from $\tilde h_+$, Φ_+ and $\tilde\Phi_-$ by replacing $\big(v \to \pm\tilde v\big)$, $\big(v_{\mp\pm} \to -(s/m)P_{\mp\pm}\beta,\, (vn) \to 0\big)$ and $\big(v_{\mp\pm} \to \mp(\tilde s/m)P_{\mp\pm}\beta,\, (vn) \to 0\big)$ respectively. The z_1-dependence of this intermediate result is cured by the twist-3 addendum given in the next section.

5.3.2. Twist-3

Out of $\mathbb{B}^{t=3}_{\mp\pm}$ the terms $\sim \ln u$ can be discarded again, they will only play a role in the helicity conserving case. Further, due to the relations (5.25) and neglecting terms with ∂_{--}, one can see that only the first term for $\mathcal{A}_{---+}$ and the second term for $\mathcal{A}_{--+-}$ in Eq. (3.39) are relevant. Therefore we are left with

$$\mathcal{A}^{t=3}_{--\mp\pm} = \frac{iz_2}{2}\int_0^1 du\, u \int_{z_2}^{z_1} \frac{dv}{z_{12}} \int \frac{d^4x\, e^{-irx}}{\pi^2 x^4}(x_{--})^2 \partial_{\mp\pm}\big[\mathcal{O}_{V,-}(uv, uz_2) \pm \mathcal{O}_{A,+}(uv, uz_2)\big] . \tag{5.43}$$

Again, effectively the calculation for h_- suffices to determine all contributions. Indeed the $\tilde{h}_+$, Φ_+, $\tilde{\Phi}_-$ parts are given by the identical replacements from the previous section.

After the Fourier transformation one can write the answer in the following form:

$$\mathcal{A}^{t=3}_{--\mp\pm}[h_-] = -2\int dh_- \left[v_{\mp\pm} + (vn)\frac{\beta P_{\mp\pm}}{(n\bar{n})}\partial_\omega\right] X(\omega)\,, \tag{5.44}$$

with

$$X(\omega) = \int_0^1 du\, u \int_{z_2}^{z_1} \frac{dv}{z_{12}} \frac{z_2}{(z_1 - uv + u\omega(v - z_2))^2} = \frac{\ln z_1}{\omega - z_1} - \frac{z_2 \ln \omega}{(\omega - 1)(\omega - z_1)} . \tag{5.45}$$

5.3.3. Summary

The twist-2 and the twist-3 answers depend on the positions individually, however in the sum this dependence disappears due to

$$W(\omega) + X(\omega) = \frac{\ln(\omega - i0)}{\omega - 1}\,, \tag{5.46}$$

as expected. Thus, the total amplitude is given by

$$\begin{aligned}
\mathcal{A}_{0\pm} = &-\frac{s}{\sqrt{2}m}\int d\Phi_+ \frac{\beta P_{\mp\pm}}{(n\bar{n})}\frac{\log(\omega - i0)}{\omega - 1} \mp \frac{\tilde{s}}{\sqrt{2}m}\int d\tilde{\Phi}_- \frac{\beta P_{\mp\pm}}{(n\bar{n})}\frac{\log(\omega - i0)}{\omega - 1} \\
&+\frac{1}{\sqrt{2}(n\bar{n})}\int dh_- \left[v_{\mp\pm} + (vn)\frac{\beta P_{\mp\pm}}{(n\bar{n})}\partial_\omega\right]\frac{\log(\omega - i0)}{\omega - 1} \\
&\pm\frac{1}{\sqrt{2}(n\bar{n})}\int d\tilde{h}_+ \left[\tilde{v}_{\mp\pm} + (\tilde{v}n)\frac{\beta P_{\mp\pm}}{(n\bar{n})}\partial_\omega\right]\frac{\log(\omega - i0)}{\omega - 1} .
\end{aligned} \tag{5.47}$$

5.4. Helicity conserving amplitudes $\mathcal{A}_{\pm\pm}$

The evaluation of the helicity conserving amplitudes $\mathcal{A}_{\pm\pm}$ was initially the main motivation for this study. Technically, they are also the most demanding ones. In order to keep matters manageable, we introduce two linear combinations

$$\mathcal{A} = \frac{1}{2}\big(\mathcal{A}_{++} + \mathcal{A}_{--}\big)\,, \qquad \tilde{\mathcal{A}} = \frac{1}{2}\big(\mathcal{A}_{++} - \mathcal{A}_{--}\big) . \tag{5.48}$$

In terms of spinor-contractions this definition reads:

$$\mathcal{A} = \frac{\mathcal{A}_{+--+} + \mathcal{A}_{-++-}}{4(n\bar{n})}\,, \quad \tilde{\mathcal{A}} = \frac{\mathcal{A}_{+--+} - \mathcal{A}_{-++-}}{4(n\bar{n})} . \tag{5.49}$$

The above separation is motivated from parity symmetry. To see its implications, let us consider the contribution of $\mathbb{B}$ to $\mathcal{A}$,

$$\mathcal{A}^{\mathbb{B}} = -\frac{1}{4(n\bar{n})}\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4}\Big\{x_{++}\big[\mathbb{B}_{--}(z_1,z_2) - \mathbb{B}_{--}(z_2,z_1)\big] + x_{--}\big[\mathbb{B}_{++}(z_1,z_2) - \mathbb{B}_{++}(z_2,z_1)\big]\Big\}\,. \tag{5.50}$$

Obviously $\mathbb{B}$ enters antisymmetrized in z_1, z_2. From parity invariance it is intuitive (and true) to suspect that only the vector part of $\mathcal{O}_{++}$ contributes. For $\mathbb{B}^{t=2}$ this is seen to hold immediately due to Eq. (3.38). In the case of $\mathbb{B}^{t=3}$ it is also clear for the terms $\sim \ln u$ of Eq. (3.39). To see that this holds for the rest of $\mathbb{B}^{t=3}$ too, one needs the following set of formulas

$$\begin{aligned}(x\bar{\Delta}\partial)_{\pm\pm} &= -(x_{\pm-}\partial_{-\pm} + \epsilon x_{\pm+}\partial_{+\pm})\,,\\ (\bar{x}\Delta\bar{\partial})_{\pm\pm} &= -(x_{-\pm}\partial_{\pm-} + \epsilon x_{+\pm}\partial_{\pm+})\,.\end{aligned} \tag{5.51}$$

From these expressions, one observes that the axial operator is preceded by terms like e.g. $(x\bar{\Delta}\partial)_{--} - (\bar{x}\Delta\bar{\partial})_{--} = \epsilon(x_{+-}\partial_{-+} - x_{-+}\partial_{+-})$, which give zero contribution after integration by parts in x. Next, consider $\mathbb{B}^{t=4}$ given in Eqs. (3.45) and (3.46). The first line of (3.46) does not require any further explanation. For the contribution from the $\mathcal{R}$- and $\bar{\mathcal{R}}$-operator, cf. Eq. (3.41), let us define for an arbitrary function f of two variables y and z

$$\begin{aligned}\mathbb{K}f &\equiv z_{12}\int_{z_2}^{z_1}\frac{dy}{z_{12}}\int_{z_2}^{y}\frac{dz}{z_{12}}\,\frac{z-z_2}{z_1-z}\,f(y,z)\,,\\ \bar{\mathbb{K}}f &\equiv z_{12}\int_{z_2}^{z_1}\frac{dy}{z_{12}}\int_{z_2}^{y}\frac{dz}{z_{12}}\,\frac{z_1-y}{y-z_2}\,f(y,z)\,.\end{aligned} \tag{5.52}$$

One easily derives

$$P_{12}\mathbb{K}P_{yz} = -\bar{\mathbb{K}}\,, \tag{5.53}$$

where P_{12} and P_{yz} are the permutation operators that exchange $z_1 \leftrightarrow z_2$ and $y \leftrightarrow z$ respectively. Therefore the antisymmetrization in $z_{1,2}$ of $\mathbb{B}^{t=4}$ translates into expressions proportional to $\mathbb{K} + P_{12}\bar{\mathbb{K}} = \mathbb{K}(1 - P_{yz})$. Thus the antisymmetrization is passed straight onto the operator[1] $\mathcal{O}_{++}$, from which again only the vector part survives. The same observation applies also to the $\mathbb{A}$ and $\mathbb{C}$ contributions.

By the same reasoning it can be shown that all contributions to $\tilde{\mathcal{A}}$ come entirely from the axial operator $\mathcal{O}_{A,+}$, all of them are exclusively through $\mathbb{B}$, since $\mathbb{A}$ and $\mathbb{C}$ enter with coefficient zero.

Further progress can be made by considering the trace of $\mathcal{A}_{\mu\nu}$ over its Lorentz indices, for which one finds using Eqs. (4.38) and (4.39)

$$\mathcal{A}^{\mu}{}_{\mu} = -\mathcal{A}_{++} - \mathcal{A}_{--} + \frac{1}{4(n\bar{n})}\mathcal{A}_{++--}\,. \tag{5.54}$$

It follows that one can write

$$\mathcal{A} = -\frac{1}{2}\mathcal{A}^{\mu}{}_{\mu} + \frac{1}{8(n\bar{n})}\big(\mathcal{A}_{++--} - \mathcal{A}_{--++}\big)\,, \tag{5.55}$$

[1] The conformal two-particle generators with $j = 1$ are symmetric under such an exchange.

where we have inserted $0 = \mathcal{A}_{--++}$ for convenience. The vanishing of $\mathcal{A}_{--++}$ has been checked in [23] up to twist-5. Let us define some abbreviations for the "sub-amplitudes"

$$\begin{aligned} \mathcal{A}_{\mathrm{Tr}} &= -\frac{1}{2}\mathcal{A}^{\mu}{}_{\mu}\,, \\ \Delta\mathcal{A} &= \mathcal{A}_{+--+} - \mathcal{A}_{-++-}\,, \\ \delta\mathcal{A} &= \mathcal{A}_{++--} - \mathcal{A}_{--++}\,, \end{aligned} \tag{5.56}$$

which we are going to work out explicitly. Translation invariance, a property expected from every amplitude, should also hold for $\mathcal{A}_{\mathrm{Tr}}$, $\Delta\mathcal{A}$ and $\delta\mathcal{A}$ individually.

5.4.1. Difference term $\delta\mathcal{A}$

At first we notice that there is no contribution from $\mathbb{B}^{t=2}$, because of a complete cancellation between $\mathcal{A}_{++--}$ and $\mathcal{A}_{--++}$ (most easily seen by integration by parts). The expansion for $\delta\mathcal{A}$ starts from twist-3 (where we can take $\Pi \approx \mathbb{1}$), and after a short calculation one arrives at

$$\begin{aligned} \delta\mathcal{A} = -\frac{i}{2}\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4}\int_0^1 du\, u \int_{z_2}^{z_1}\frac{dv}{z_{12}}\times \\ \times\Big\{\Big[x_{+-}\big((x\bar{\Delta}\partial)_{-+} - (\bar{x}\Delta\bar{\partial})_{+-}\big) + x_{-+}\big((\bar{x}\Delta\bar{\partial})_{-+} - (x\bar{\Delta}\partial)_{+-}\big)\Big]\times \\ \times\Big[z_1 O_{V,-}(uz_1, uv) - z_2 O_{V,-}(uv, uz_2)\Big]\Big\}\,. \end{aligned} \tag{5.57}$$

A little algebra using Eq. (5.25) and integration by parts in each of the resulting eight terms reveals that the square brackets in the second line of (5.57) can by written as

$$-16(n\bar{n})\big((x\bar{n}) - \epsilon(xn)\big) - 4ix_{+-}x_{-+}(q\Delta) \approx -16(n\bar{n})\big((x\bar{n}) - \epsilon(xn)\big) + 4ix_{+-}x_{-+}(n\bar{n})\,. \tag{5.58}$$

Inserting the DD parametrization for the matrix element of $\mathcal{O}_{V,+}$ one gets for the contribution from h_-

$$\begin{aligned} \delta\mathcal{A}^{t=3}[h_-] = 8i(n\bar{n})\int dh_- \int_0^1 du\, u\int_{z_2}^{z_1}\frac{dv}{z_{12}}\times \\ \times\int\frac{d^4x\, e^{-irx}}{\pi^2x^4}\left((x\bar{n}) - \epsilon(xn) + \frac{x_{+-}x_{-+}}{4i}\right)(vx)\Big(z_1 e^{-iu\ell_{1v}x} - z_2 e^{-iu\ell_{v2}x}\Big). \end{aligned} \tag{5.59}$$

Taking the momentum space integral yields

$$\begin{aligned} \delta\mathcal{A}^{t=3}[h_-] = 8\int dh_- \Big[\epsilon(vn)\Big(2 + \frac{\beta}{\xi}\partial_\omega\Big) - \frac{v_{+-}P_{-+} + v_{-+}P_{+-}}{4(n\bar{n})}\beta\partial_\omega \\ - \frac{(vn)P_{+-}P_{-+}}{4(n\bar{n})^2}\beta^2\partial_\omega^2\Big]\frac{\ln(\omega - i0)}{\omega - 1}\,. \end{aligned} \tag{5.60}$$

As one might have expected, it was possible to express the u- and v-integration by V_-, see Eqs. (5.29), (5.34). A small simplification can be achieved by

$$v_{+-}P_{-+} + v_{-+}P_{+-} = -4(n\bar{n})\big((vn)\epsilon/\xi + (vP)\big)\,, \tag{5.61}$$

along with $\epsilon = t/(2(n\bar{n}))$ and $P_{+-}P_{-+} = 2(n\bar{n})|P_\perp|^2$, which allows us to rewrite the expression for $\delta\mathcal{A}^{t=3}[h_-]$ as

$$\delta\mathcal{A}^{t=3}[h_-] = 8\int dh_-\left[\frac{(vn)t}{(n\bar{n})}\left(1+\frac{\beta}{\xi}\partial_\omega\right) + (vP)\beta\partial_\omega - \frac{(vn)|P_\perp|^2}{2(n\bar{n})}\beta^2\partial_\omega^2\right]\frac{\ln(\omega - i0)}{\omega - 1}. \quad (5.62)$$

The contribution from Φ_+ is calculated analogously:

$$\delta\mathcal{A}^{t=3}[\Phi_+] = \frac{4(n\bar{n})s}{m}\int d\Phi_+\left[\frac{t}{\xi(n\bar{n})} + \frac{|P_\perp|^2}{2(n\bar{n})}\beta^2\partial_\omega^2\right]\frac{\ln(\omega - i0)}{\omega - 1}. \quad (5.63)$$

Next, one notices that $\mathbb{B}^{t=4}$ does not contribute to $\delta\mathcal{A}$. This can be verified for example by integration by parts along with the symmetry properties given at the beginning of Sec. 5.4. The other twist-4 structure, $\mathbb{A}$, is trivial since it enters with coefficient zero. Last, the only contribution from the twist-4 sector comes from $\mathbb{C}$ and reads

$$\delta\mathcal{A}^{t=4} = -2\int \frac{d^4x\, e^{-irx}}{\pi^2 x^2}\big(x_{--}\partial_{++} - x_{++}\partial_{--}\big)\big(\mathbb{C}(z_1,z_2) + \mathbb{C}(z_2,z_1)\big)\,. \quad (5.64)$$

Now, since $x_{++}\partial_{--}$ gives only a subleading (beyond twist-4) correction, it can be discarded. For the same reason, the difference $x_{--}\partial_{++} - 4(n\bar{n})(x\partial)$ is of higher order (than twist-4). One can therefore replace $x_{--}\partial_{++}$ by $4(n\bar{n})(x\partial)$ and write

$$\delta\mathcal{A}^{t=4} = -8(n\bar{n})\int \frac{d^4x\, e^{-irx}}{\pi^2 x^2}(x\partial)\big(\mathbb{C}(z_1,z_2) + \mathbb{C}(z_2,z_1)\big)\,, \quad (5.65)$$

which is correct to our accuracy. We postpone dealing with this expression to the trace part, and combine it with a similar integral in that context.

5.4.2. Trace part $\mathcal{A}_{\mathrm{Tr}}$

In the spinor formalism the term proportional to the trace of $\mathcal{A}_{\mu\nu}$ can be expressed as

$$\mathcal{A}_{\mathrm{Tr}} = \frac{1}{4}\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\mathcal{A}_{\alpha\dot\alpha\beta\dot\beta}\,. \quad (5.66)$$

From $\mathbb{B}$ we get,

$$\mathcal{A}_{\mathrm{Tr}}^{\mathbb{B}} = -\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4}\big((x\mathbb{B})(z_1,z_2) - (x\mathbb{B})(z_2,z_1)\big)\,, \quad (5.67)$$

with $(x\mathbb{B})(z_1,z_2) = x^\mu \mathbb{B}_\mu(z_1,z_2)$ and $\mathbb{B}_\mu(z_1,z_2)$ being the Lorentz-vector associated to $\mathbb{B}_{\alpha\dot\alpha}$, see Eq. (3.26).

In particular for twist-2 one has

$$\mathcal{A}_{\mathrm{Tr}}^{t=2} = -\int_0^1 du\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4}(x\partial)\mathcal{O}^{t=2}_{V,-}(uz_1,uz_2)\,. \quad (5.68)$$

An immediate simplification is achieved by the observation[2]

$$u\partial_u\big(\bar{q}(uz_1x)u\not{x}q(uz_2x)\big) = (x\partial)\big(\bar{q}(uz_1x)u\not{x}q(uz_2x)\big)\,. \quad (5.69)$$

[2]Roughly speaking, "$(x\partial)$ counts the degree of $\mathcal{O}_V$ in x, which is one more than the degree in u".

The operator in the parentheses of this equation is equal to $u\mathcal{O}_V(z_1u, z_2u)$, allowing us to replace $(x\partial)$ by $\partial_u u$ in Eq. (5.68). This property is neither altered by antisymmetrization of the arguments nor by the leading twist projection, which does not change the degree in x or u. Then the integration over u trivializes and yields

$$\mathcal{A}^{t=2}_{\rm Tr} = -\int \frac{d^4x\, e^{-irx}}{\pi^2 x^4}\mathcal{O}^{t=2}_{V,-}(z_1, z_2)\,. \tag{5.70}$$

It is at this point where we have to take into account the projector Π beyond trivial order for the first time. Instead of using Eq. (3.33) directly, we have found it more convenient to work with the representation (A.11). Although the former approach would work perfectly well, it produces extremely long and cumbersome expressions in intermediate steps.

Doing the Fourier integral yields for h_-

$$\mathcal{A}^{t=2}_{\rm Tr}[h_-] = 2\int dh_-\left[\frac{(v, r+\ell_{12})}{(r+\ell_{12})^2} + \int_0^1 du\, u\left[\frac{u\ell_{12}^2(v, r+u\ell_{12})}{(r+u\ell_{12})^4} - \frac{(v\ell_{12})}{(r+u\ell_{12})^2}\right]\right]. \tag{5.71}$$

The term under the u-integral comes from the $\mathcal{O}(x^2)$ expansion of Π and gives rise to power corrections due to its prefactors $\ell_{12}^2 \sim \mathcal{O}(m^2, t)$ and $(v\ell_{12}) = -\beta(vP) = -\beta ms \sim \mathcal{O}(m^2, m\sqrt{t})$. For this part it is adequate to use the approximate formula (5.20) and in the numerator, $(v, r+u\ell_{12}) = (vn) - u\beta z_{12}(vP) \approx (vn)$. For the first term however, one needs to keep the first order power corrections in parallel. That means keeping the exact expression $(v, r+\ell_{12})$ as well as expanding the denominator to the first order in t and $|P_\perp|^2$, which reads (under the condition $z_{12} = 1$)

$$(r+\ell_{12})^2 = -2(n\bar{n})\omega + t\big(\alpha\omega - \omega(\omega-1)\big) - |P_\perp|^2\beta^2\,. \tag{5.72}$$

After a little calculation we obtain

$$\begin{aligned}\mathcal{A}^{t=2}_{\rm Tr}[h_-] = \int dh_-\Bigg\{\frac{(vn)}{(n\bar{n})}\Bigg[&-\left(\partial_\omega + \frac{|P_\perp|^2\beta^2}{2(n\bar{n})}\partial_\omega^2 - \frac{t}{2(n\bar{n})}\frac{\beta}{\xi}\partial_\omega\right)\ln(\omega - i0)\\ &-\left(\frac{|P_\perp|^2\beta^2}{2(n\bar{n})} + \frac{t}{2(n\bar{n})}(\omega - z_1)(\alpha - \omega - z_2)\right)\partial_\omega^2\, U(\omega)\Bigg]\\ &+\frac{(vP)\beta}{(n\bar{n})}\partial_\omega\big(\ln(\omega - i0) - U(\omega)\big)\Bigg\}\,.\end{aligned} \tag{5.73}$$

As in the previous cases we merely quote the result for Φ_+:

$$\begin{aligned}\mathcal{A}^{t=2}_{\rm Tr}[\Phi_+] = \frac{s}{m}\int d\Phi_+\Bigg\{&\left(1 + \frac{|P_\perp|^2\beta^2}{2(n\bar{n})}\right)\ln(\omega - i0)\\ &+\left(-\frac{|P_\perp|^2\beta^2}{2(n\bar{n})} + \frac{t}{2(n\bar{n})}(\omega - z_1)(\alpha - \omega - z_2)\right)\partial_\omega\, U(\omega)\Bigg\}\,.\end{aligned} \tag{5.74}$$

The twist-3 part for the trace is actually trivial. Consider e.g. the contribution stemming from the term with prefactor z_1 in Eq. (3.39). It is proportional to

$$\int_0^1 du\, u\, \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\big(x_{\alpha\dot\alpha}(x\bar{\Delta}\partial)_{\beta\dot\beta} + x_{\alpha\dot\alpha}\partial_{\beta\dot\beta}x^2\ln u(\Delta\partial)\big)\mathcal{O}^{t=2}_{V,-}(uz_1, uv)\,. \tag{5.75}$$

The differential operator in the parentheses can be simplified to

$$-2x^2(\Delta\partial) - 2\ln u(x\partial)x^2(\Delta\partial)\,. \tag{5.76}$$

Here an analogous "trick" as in the beginning of this section is applicable, representing $(x\partial) \to u^{-1}\partial_u u^2$. Integration by parts in u with vanishing boundaries cancels the first term in the above equation. The same is true for the z_2-proportional term in Eq. (3.39). Thus

$$(x\mathbb{B}^{t=3}) = 0\,. \tag{5.77}$$

i.e. there is no twist-3 contribution to the trace $\mathcal{A}_{\rm Tr}$.

In the rest of this section we will be dealing with the twist-4 sector, starting with $\mathbb{C}$, Eq. (3.47). Taking the trace yields a contribution of the form

$$\int \frac{d^4x\, e^{-irx}}{\pi^2 x^2}(x\partial)\big(\mathbb{C}(z_1,z_2) - \mathbb{C}(z_2,z_1)\big)\,. \tag{5.78}$$

Recall, that we had a similar term in the evaluation of $\delta\mathcal{A}$, see Eq. (5.65). The sum of the above contribution and (5.65), with the factor of $8(n\bar{n})$ removed, will be called $\mathcal{A}^{\mathbb{C}}_{\rm Tr}$ and is given by

$$\mathcal{A}^{\mathbb{C}}_{\rm Tr} = -2\int \frac{d^4x\, e^{-irx}}{\pi^2 x^2}(x\partial)\mathbb{C}(z_2,z_1)\,. \tag{5.79}$$

Note the "reverse ordering" of z_i in this expression. Here, $\mathbb{C}$ is written as

$$\mathbb{C}(z_1,z_2) = -\frac{1}{4}\int_0^1 \frac{du}{u^2}\,\mathcal{R}_V(uz_1,uz_2)\,, \tag{5.80}$$

where $\mathcal{R}_V$ is given by $\mathcal{R}$ in Eq. (3.41) with $\mathcal{O}^{t=2}_{++}$ replaced by $\mathcal{O}^{t=2}_{V,-}$. In more detail

$$\mathcal{R}_V(z_1,z_2) = -\mathbb{K}\left(\frac{t}{2}S_+ + i(\Delta\partial)\right)\mathcal{O}^{t=2}_{V,+}\,. \tag{5.81}$$

By similar arguments that led to Eq. (5.69), it is easy to find the following scaling behavior of $\mathcal{R}_V$,

$$(x\partial)\mathcal{R}_V(uz_1,uz_2) = u^2\partial_u\frac{1}{u}\mathcal{R}_V(uz_1,uz_2)\,, \tag{5.82}$$

which simplifies $\mathcal{A}^{\mathbb{C}}_{\rm Tr}$ by direct integration in u to

$$\mathcal{A}^{\mathbb{C}}_{\rm Tr} = \int \frac{d^4x\, e^{-irx}}{2\pi^2 x^2}\mathcal{R}_V(z_2,z_1)\,. \tag{5.83}$$

In order to arrive at this result, it was used that $\mathcal{R}_V(uz_1,uz_2)$ vanishes at least quadratically as $u\to 0$. Instead of evaluating $\mathcal{A}^{\mathbb{C}}_{\rm Tr}$ directly, we generalize it to an expression of much use for the later part of the calculation. Let

$$\mathfrak{R}(z_1,z_2|z_1^*) \equiv \int \frac{d^4x\, e^{-ir^*x}}{2\pi^2 x^2}\mathcal{R}_V(z_1,z_2) \tag{5.84}$$

with

$$r^* = n + z_1^*\Delta\,. \tag{5.85}$$

Due to technical reasons and re-usability in a later calculation, we do *not* assume $z_{12} = 1$, $z_1^* = z_1$ in $\mathfrak{R}(z_1, z_2|z_1^*)$. For convenience let us also focus on the h_- part of $\mathcal{O}_V$ and call it $\mathfrak{R}_{h_-}(z_1, z_2|z_1^*)$. Let us denote the relevant parts from $\mathcal{O}_1$ and $\mathcal{O}_2$ as $\mathcal{O}_{1,h}$ and $\mathcal{O}_{2,h}$ respectively:

$$\begin{aligned}
\mathcal{O}_{1,h}(y,z) &= -t\int dh_-\,(vx)e^{-i\ell_{yz}x}\,,\\
\mathcal{O}_{2,h}(y,z) &= \int dh_-\Big[(vx)(\Delta\ell_{yz})e^{-i\ell_{yz}x} + i\frac{(x\Delta)}{2}\int_0^1 du\,\big(u^2(vx)\ell_{yz}^2 + 2iu(v\ell_{yz})\big)e^{-iu\ell_{yz}x}\Big]\,.
\end{aligned} \tag{5.86}$$

The contribution from $\mathcal{O}_{1,h}$ already had a prefactor of t to begin with, so the leading twist projection can be taken as unity. This is not the case in $\mathcal{O}_{2,h}$, where the $\mathcal{O}(x^2)$ correction of Π has to be taken into account. Luckily, the additional u-integration can be taken immediately in the construction of $\mathfrak{R}_{h_-}(z_1, z_2|z_1^*)$ by means of

$$(S_0 - 1)\int_0^1 du\big(u^2(vx)\ell_{yz}^2 + 2iu(v\ell_{yz})\big)e^{-iu\ell_{yz}x} = \big((vx)\ell_{yz}^2 + 2i(v\ell_{yz})\big)e^{-i\ell_{yz}x}\,. \tag{5.87}$$

Thus we get

$$\begin{aligned}
\mathfrak{R}_{h_-}(z_1, z_2|z_1^*) = -\int dh_-\,\mathbb{K}\int \frac{d^4x\,e^{-ir^*x}}{\pi^2x^2}\Big[(vx)\Big(\frac{t}{2}S_+ + (S_0-1)(\Delta\ell_{yz})\\
+\frac{i}{2}(x\Delta)\ell_{yz}^2\Big) - (\ell_{yz}v)(x\Delta)\Big]e^{-i\ell_{yz}x}\,,
\end{aligned} \tag{5.88}$$

After the Fourier transformation, under the usual approximations, we obtain

$$\begin{aligned}
\mathfrak{R}_{h_-}(z_1, z_2|z_1^*) = 8\int dh_-\,\mathbb{K}\Big[\Big(\frac{t}{2}S_+ + (S_0-1)(\Delta\ell_{yz})\Big)\frac{(vn)}{(r^*+\ell_{yz})^4}\\
+\frac{(n\bar{n})(v\ell_{yz})}{(r^*+\ell_{yz})^4} - 2\frac{(n\bar{n})(vn)\ell_{yz}^2}{(r^*+\ell_{yz})^6}\Big]\,,
\end{aligned} \tag{5.89}$$

and by using Eq. (5.20) with $z_1\bar{u} \to z_1^*$ and $z_{12}u \to y - z$ for the denominators one gets

$$\begin{aligned}
\mathfrak{R}_{h_-}(z_1, z_2|z_1^*) = 2\int dh_-\Big[\frac{(vP)\beta}{(n\bar{n})}\partial_\omega - \frac{(vn)|P_\perp|^2\beta^2}{2(n\bar{n})^2}\partial_\omega^2\\
-\frac{(vn)t}{2(n\bar{n})}\big(\omega(\omega-1)\partial_\omega^2 + 2\alpha\partial_\omega\big)\Big]\mathfrak{I}(z_1, z_2|z_1^*).
\end{aligned} \tag{5.90}$$

Here

$$\mathfrak{I}(z_1, z_2|z_1^*) = \mathbb{K}\frac{1}{\omega(y-z) - y + z_1^* - i0}\,. \tag{5.91}$$

The evaluation of $\mathfrak{I}$ is in principle elementary but in full generality rather lengthy. It can be written as

$$\mathfrak{I}(z_1, z_2|z_1^*) = \frac{1}{\omega - 1}\int_{z_2}^{z_1} \frac{dz}{z_{12}}\,\frac{z - z_2}{z_1 - z}\ln\!\left(\frac{\omega(z_1 - z) + z_1^* - z_1 - i0}{z_1^* - z}\right). \tag{5.92}$$

For the case at hand it yields

$$\mathfrak{I}(z_2, z_1|z_1^* = z_1)\Big|_{z_{12}=1} = -\frac{\ln(1-\omega - i0)}{\omega} - \frac{\mathrm{Li}_2(\omega + i0) - \mathrm{Li}_2(1)}{\omega - 1}. \tag{5.93}$$

The dilogarithm (Spence function) entering here is defined by

$$\mathrm{Li}_2(z) = -\int_0^z \frac{dy}{y}\log(1-y)\,, \quad \mathrm{Li}_2(1) = \frac{\pi^2}{6}\,, \tag{5.94}$$

one needs to keep the $i0$ due to the logarithmic branching point of $\mathrm{Li}_2(z)$ at $z = 1$. Collecting everything we have obtained

$$\mathcal{A}^{\mathbb{C}}_{\mathrm{Tr}}[h_-] = \int dh_-\left[\frac{(vn)|P_\perp|^2\beta^2}{2(n\bar{n})^2}\partial_\omega + \frac{(vn)t}{2(n\bar{n})}\big(\omega(\omega-1)\partial_\omega + 2\alpha\big) - \frac{(vP)\beta}{(n\bar{n})}\right]\times$$
$$\times\partial_\omega\left[\frac{\mathrm{Li}_2(\omega+i0) - \mathrm{Li}_2(1)}{\omega - 1} + \frac{\log(1-\omega-i0)}{\omega}\right], \tag{5.95}$$

and, analogously

$$\mathcal{A}^{\mathbb{C}}_{\mathrm{Tr}}[\Phi_+] = \frac{s}{2m}\int d\Phi_+\left[\frac{|P_\perp|^2\beta^2}{(n\bar{n})}\partial_\omega + \frac{t}{(n\bar{n})}\big(\omega(\omega-1)\partial_\omega + \alpha\big)\right]\times$$
$$\times\left[\frac{\mathrm{Li}_2(\omega+i0) - \mathrm{Li}_2(1)}{\omega - 1} + \frac{\log(1-\omega-i0)}{\omega}\right]. \tag{5.96}$$

Apparently $\mathcal{A}^{\mathbb{C}}_{\mathrm{Tr}}$ does not depend on z_1, which is a consequence of the fact that $\mathcal{R}(z_1, z_2)$ is already translation invariant up to twist-6 corrections,

$$(\partial_{z_1} + \partial_{z_2})\mathcal{R}(z_1, z_2) = \big[i(\mathbf{P}x), \mathcal{R}(z_1, z_2)\big] + \text{twist-6}\,. \tag{5.97}$$

This relation can be proven by direct computation. The operator on the l.h.s. is given by the "step-down" operator of $SL(2,\mathbb{R})$

$$S_-^{(j_1,j_2)} = -\partial_{z_1} - \partial_{z_2}\,, \tag{5.98}$$

which satisfies the following commutation relations with the other generators $S_0^{(j_1,j_2)}, S_+^{(j_1,j_2)}$ in Eq. (3.44)

$$\Big[S_+^{(j_1,j_2)}, S_-^{(j_1,j_2)}\Big] = 2S_0^{(j_1,j_2)}\,, \quad \Big[S_0^{(j_1,j_2)}, S_\pm^{(j_1,j_2)}\Big] = \pm S_\pm^{(j_1,j_2)}\,. \tag{5.99}$$

We recall the definition of $\mathcal{R}(z_1, z_2)$

$$\mathcal{R}(z_1, z_2) = \mathbb{K}\bigg(\frac{1}{2}S_+^{(1,1)}\big[i\mathbf{P}^\mu, \big[i\mathbf{P}_\mu, \mathcal{O}^{t=2}_{++}\big]\big] - (S_0^{(1,1)} - 1)\big[i\mathbf{P}^\mu, \partial_\mu\mathcal{O}^{t=2}_{++}\big]\bigg). \tag{5.100}$$

Our strategy is to move $S_-^{(j_1,j_2)}$ to the right until it "hits" the fundamental fields through the operator $\mathcal{O}_{++}$, where

$$-S_-^{(j_1,j_2)}\mathcal{O}_{++}(z_1,z_2) = \big[i(\mathbf{P}n),\mathcal{O}_{++}\big]\,. \tag{5.101}$$

The first step is to notice that $\mathbb{K}$ is an intertwining operator for the representations $\left(\frac{3}{2},\frac{1}{2}\right)$ and $(1,1)$, i.e.

$$S_{0,\pm}^{(\frac{3}{2},\frac{1}{2})}\mathbb{K} = \mathbb{K}S_{0,\pm}^{(1,1)}\,. \tag{5.102}$$

Note that on the l.h.s. of this equation the generators act on the variables z_1, z_2 and on the r.h.s. they act inside the integral on the integration variables y, z in Eq. (5.52). Of course, Eq. (5.102) can be verified directly. In a more general framework relations such as (5.102) are presented in [35]. Since $S_-^{(j_1,j_2)}$ does not depend on the conformal spins it is simply passed through the kernel $\mathbb{K}$. Then we obtain using (5.99)

$$\begin{aligned}(\partial_{z_1}+\partial_{z_2})\mathcal{R}(z_1,z_2) = -\mathbb{K}\bigg[\bigg(\frac{1}{2}S_+^{(1,1)}S_-^{(1,1)} - S_0^{(1,1)}\bigg)\big[i\mathbf{P}^\mu,\big[i\mathbf{P}_\mu,\mathcal{O}_{++}^{t=2}\big]\big]&\\ - S_0^{(1,1)}S_-^{(1,1)}\big[i\mathbf{P}^\mu,\partial_\mu\mathcal{O}_{++}^{t=2}\big]\bigg]\,.&\end{aligned} \tag{5.103}$$

At this point it is important to realize that $S_-^{(1,1)}$ does not commute with the leading twist projection, basically due to the presence of an additional n in Eq. (5.101). Instead one has the "product rule" (A.12)

$$-S_-^{(1,1)}\mathcal{O}_{++}^{t=2}(y,z) = \big[i(\mathbf{P}x),\mathcal{O}_{++}^{t=2}(y,z)\big] - \frac{1}{2}x^2\int_0^1 du\,u\,\mathcal{O}_2(uy,uz)\,. \tag{5.104}$$

It is then easy to derive for $\mathcal{O}_1 = \big[i\mathbf{P}^\mu,\big[i\mathbf{P}_\mu,\mathcal{O}_{++}^{t=2}\big]\big]$ and $\mathcal{O}_2 = \big[i\mathbf{P}^\mu,\partial_\mu\mathcal{O}_{++}^{t=2}\big]$

$$\begin{aligned}-S_-^{(1,1)}\mathcal{O}_1(y,z) &= \big[i(\mathbf{P}x),\mathcal{O}_1(y,z)\big] + \text{twist-6}\,,\\ -S_-^{(1,1)}\mathcal{O}_2(y,z) &= \big[i(\mathbf{P}x),\mathcal{O}_2(y,z)\big] + \mathcal{O}_1(y,z) - \int_0^1 du\,u\,\big[i(\mathbf{P}x),\mathcal{O}_2(uy,uz)\big] + \text{twist-6}\,.\end{aligned} \tag{5.105}$$

In the above equations, "twist-6" stands for contributions of operators of the type

$$\big[i\mathbf{P}_\mu,\big[i\mathbf{P}^\mu,\big[i\mathbf{P}_\nu,\partial^\nu\mathcal{O}_{++}^{t=2}\big]\big]\big] \quad\text{and}\quad \big[i\mathbf{P}_\mu,\partial^\mu\big[i\mathbf{P}_\nu,\partial^\nu\mathcal{O}_{++}^{t=2}\big]\big]\,, \tag{5.106}$$

which can be neglected. We can now collect everything, which yields

$$\begin{aligned}(\partial_{z_1}+\partial_{z_2})\mathcal{R}(z_1,z_2) = \mathbb{K}\bigg(\frac{1}{2}S_+^{(1,1)}\big[i(\mathbf{P}x),\mathcal{O}_1(y,z)\big] - S_0^{(1,1)}\big[i(\mathbf{P}x),\mathcal{O}_2(y,z)\big]&\\ + S_0^{(1,1)}\int_0^1 du\,u\,\big[i(\mathbf{P}x),\mathcal{O}_2(uy,uz)\big]\bigg) + \text{twist-6}\,.&\end{aligned} \tag{5.107}$$

Now the action of $S_0^{(1,1)} = y\partial_y + z\partial_z + 2$ can be traded for $\frac{1}{u}\partial_u u^2$ under the integral in the second line. Direct integration produces simply $\big[i(\mathbf{P}x),\mathcal{O}_2(y,z)\big]$. Now the commutator $[i(\mathbf{P}x),\ldots]$, can be "pulled" to the left, thus establishing Eq. (5.97).

Next, we turn back to the amplitude $\mathcal{A}_{\mathrm{Tr}}$ and consider the contributions of $\mathbb{A}$ and $\mathbb{B}^{t=4}$, which can be conveniently combined into

$$\mathcal{A}_{\rm Tr}^{\mathbb{A}+\mathbb{B}^{t=4}} = -2\int \frac{d^4x\, e^{-irx}}{x^2\pi^2}\big[\mathbb{A}(z_1,z_2)+(x\partial)\mathbb{B}^{t=4}(z_1,z_2)\big]\,. \tag{5.108}$$

The action of the differential operator $(x\partial)$ can again be traded for one in u in Eq. (3.46), and after integration by parts, everything can be compactly written as

$$\mathcal{A}_{\rm Tr}^{\mathbb{A}+\mathbb{B}^{t=4}} = \frac{1}{4}\int \frac{d^4x\, e^{-irx}}{x^2\pi^2}\int_0^1 du\,\Big[tz_1z_2u^2\mathcal{O}_{V,-}(uz_1,uz_2)+(1-P_{12})z_2\partial_{z_2}^2 z_{12}\mathcal{R}_V(uz_1,uz_2)\Big]. \tag{5.109}$$

Here P_{12} denotes the permutation operator, exchanging z_1 with z_2 and vice versa. The first expression, up to the contribution of Φ_+,

$$\frac{1}{4}\int \frac{d^4x\, e^{-irx}}{x^2\pi^2}\int_0^1 du\, tz_1z_2u^2O_{V,-}(uz_1,uz_2) = \int dh_-\,\frac{(vn)t}{2(n\bar n)^2}z_1z_2\partial_\omega^2 U(\omega)+\ldots \tag{5.110}$$

cancels vs. the z_1z_2-proportional term from the twist-2 answer in Eq. (5.73). The second one reads, in the notation introduced before,

$$\begin{aligned}&\int dh_-\left[\frac{(vP)\beta}{(n\bar n)}-\frac{(vn)|P_\perp|^2\beta^2}{2(n\bar n)^2}\partial_\omega-\frac{(vn)t}{2(n\bar n)}\big(\omega(\omega-1)\partial_\omega+2\alpha\big)\right]\partial_\omega\times\\ &\quad\times\frac{1}{2}\int_0^1 du\,(1-P_{12})z_2\partial_{z_2}^2 z_{12}\mathfrak{I}(uz_1,uz_2|z_1^*)\,,\end{aligned} \tag{5.111}$$

with $z_1^*=z_1$ in the end and

$$z_2\partial_{z_2}^2 z_{12}\mathfrak{I}(z_1,z_2|z_1^*) = \frac{1}{\omega-1}\frac{z_2}{z_{12}}\ln\!\left(\frac{z_{12}\omega+z_1^*-z_1-i0}{z_1^*-z_2}\right). \tag{5.112}$$

Note that in order to bring this into the form required by Eq. (5.111), one can simply rescale $z_1\to uz_1$, $z_2\to uz_2$ in the above expression, followed by antisymmetrization in z_1 and z_2. After that we can set $z_{12}=1$ and $z_1^*=z_1$. Integration over u gives for the second line of Eq. (5.111)

$$\frac{1}{2}\left[\frac{(z_1+z_2)\ln(\omega-i0)}{\omega-z_1}-\frac{2\ln z_1}{\omega-z_1}+\frac{z_2\ln(\omega-i0)}{(\omega-1)(\omega-z_1)}\right]. \tag{5.113}$$

In order to arrive at this result, it was implicitly used in intermediate steps that h_- is antisymmetric w.r.t. the transformation $(\beta,\alpha)\to(-\beta,-\alpha)$. Last, it remains to add up all terms. Translation invariance is restored in the sum of $\mathcal{A}_{\rm Tr}^{t=2}$ and $\mathcal{A}_{\rm Tr}^{\mathbb{A}+\mathbb{B}^{t=4}}$ by means of

$$\frac{1}{2}\int_0^1 du\,(1-P_{12})z_2\partial_{z_2}^2 z_{12}\mathfrak{I}(uz_1,uz_2|z_1^*)-U(\omega) = \frac{(1-2\omega)\ln(\omega-i0)}{2(\omega-1)}\,. \tag{5.114}$$

Collecting all contributions in $\mathcal{A} = \mathcal{A}_{\mathrm{Tr}} + \delta\mathcal{A}$ one gets (eliminating α by $\alpha = 2\omega - \beta/\xi - 1$)

$$\begin{aligned} \mathcal{A}[h_-] = -\int dh_- \Bigg\{ \frac{(vn)}{(n\bar{n})}\frac{1}{\omega - i0} - \Bigg[\frac{(vn)|P_\perp|^2\beta^2}{2(n\bar{n})^2}\partial_\omega^2 - \frac{\beta(vP)}{(n\bar{n})}\partial_\omega \\ + \frac{(vn)t}{(n\bar{n})^2}\left(\frac{1}{2}\partial_\omega(\omega-1) - \frac{\beta}{\xi}\partial_\omega - 1\right)\Bigg]\left[\frac{1}{2}\frac{\log(\omega - i0)}{\omega - 1} + \frac{\mathrm{Li}_2(\omega + i0) - \mathrm{Li}_2(1)}{\omega - 1}\right]\Bigg\}. \end{aligned} \tag{5.115}$$

The analogous contribution from the Φ_+-sector reads

$$\begin{aligned} \mathcal{A}[\Phi_+] = \frac{s}{m}\int d\Phi_+ \Bigg\{ \log(\omega - i0) - \left[\frac{|P_\perp|^2\beta^2}{2(n\bar{n})}\partial_\omega + \frac{t}{2(n\bar{n})}\left(\omega - \frac{\beta}{\xi} - 1\right)\right]\times \\ \times\left[\frac{1}{2}\frac{\log(\omega - i0)}{\omega - 1} + \frac{\mathrm{Li}_2(\omega + i0) - \mathrm{Li}_2(1)}{\omega - 1}\right]\Bigg\}. \end{aligned} \tag{5.116}$$

This is a remarkably compact result compared to the intermediate expressions that led to it.

5.4.3. Helicity difference $\Delta\mathcal{A}$

In order to obtain the answer for $\Delta\mathcal{A} = \frac{1}{2}(\mathcal{A}_{++} - \mathcal{A}_{--})$ an independent calculation is required. Given that the calculation in the previous sections was very detailed, we skip the technical details and only flash intermediate results.

From $\mathbb{B}^{t=2}$ one gets

$$\Delta\mathcal{A}^{t=2} = \frac{i}{(n\bar{n})}\int_0^1 du \int \frac{d^4x e^{-irx}}{\pi^2 x^4}\big((r\bar{n})(xn) - (rn)(x\bar{n})\big)\mathcal{O}^{t=2}_{A,+}(uz_1, uz_2)\,, \tag{5.117}$$

where one needs the projector Π including its $\mathcal{O}(x^2)$ correction. It results in

$$\begin{aligned} \Delta\mathcal{A}^{t=2}[\tilde{\Phi}_-] = \frac{\tilde{s}}{m}\int d\tilde{\Phi}_- \Bigg\{ \ln(\omega - i0) + \frac{1}{2}\Bigg[\frac{|P_\perp|^2\beta^2}{(n\bar{n})}(\omega - z_1)\partial_\omega^2 \\ + \frac{t}{(n\bar{n})}\left(\omega - z_1 - \frac{\beta}{\xi}\right)\big((\omega - z_1)\partial_\omega - 1\big)\Bigg]\frac{z_1\,\mathrm{Li}_2(1 - \omega/z_1 + i0)}{\omega - z_1}\Bigg\} \end{aligned} \tag{5.118}$$

and

$$\begin{aligned} \Delta\mathcal{A}^{t=2}[\tilde{h}_+] = \int d\tilde{h}_+ \Bigg\{ \frac{-1}{\omega - i0} + \Bigg[\frac{(\tilde{v}n)|P_\perp|^2\beta^2}{2(n\bar{n})^2}\partial_\omega^2 - \frac{(\tilde{v}n)t}{(n\bar{n})^2}\left(1 - \frac{\beta}{\xi}\partial_\omega\right) \\ - \frac{(\tilde{v}\Delta)}{(n\bar{n})}\left(1 + \frac{\beta}{2\xi}\right)\partial_\omega\Bigg]\big(1 - (\omega - z_1)\partial_\omega\big)\frac{z_1\,\mathrm{Li}_2(1 - \omega/z_1 + i0)}{\omega - z_1} \\ + \frac{(\tilde{v}n)t}{(n\bar{n})^2}\left[\frac{z_1}{\omega - i0} - \frac{z_1\ln(\omega/z_1 - i0)}{\omega - z_1}\right]\Bigg\} \end{aligned} \tag{5.119}$$

Unfortunately, $(x\mathbb{B}^{t=3}) = 0$ is of no help in the twist-3 sector and one needs to calculate

$$\Delta\mathcal{A}^{t=3} = \int_0^1 du\, u \int_{z_2}^{z_1} \frac{dv}{z_{12}} \int \frac{d^4x\, e^{-irx}}{\pi^2 x^4} \left[(xn)(x\bar{n}) - \frac{\ln u}{2} x^2 (x, n + z_1\bar{n})(\Delta\partial)\right] \times$$
$$\times \left[z_1 \mathcal{O}_{A,+}(uz_1, uv) + z_2 \mathcal{O}_{A,+}(uv, uz_2)\right]. \qquad (5.120)$$

After a rather long calculation, where Π has also to be taken in next-to-trivial order, we get

$$\Delta\mathcal{A}^{t=3}[\tilde{\Phi}_-] = \frac{\tilde{s}}{m} \int d\tilde{\Phi}_- \left[\frac{|P_\perp|^2\beta^2}{(n\bar{n})}\partial_\omega - \frac{t}{(n\bar{n})}\frac{\beta}{\xi}\right]\left[\frac{\ln(\omega - i0)}{2(\omega-1)} - \frac{z_1\ln(\omega/z_1 - i0)}{\omega - z_1}\right.$$
$$\left. + \frac{z_1 \operatorname{Li}_2(1-\omega/z_1 + i0)}{\omega - z_1}\left(\frac{z_1}{\omega} + \frac{z_2}{\omega - 1}\right) + \frac{z_1\big(\operatorname{Li}_2(1) + \operatorname{Li}_2(z_2/z_1)\big)}{\omega - i0}\right] \qquad (5.121)$$

and

$$\Delta\mathcal{A}^{t=3}[\tilde{h}_+] = \frac{(\tilde{v}n)}{(n\bar{n})} \int d\tilde{h}_+ \left[\frac{2t}{(n\bar{n})}\left(1 + \frac{\beta}{\xi}\partial_\omega\right) - \frac{|P_\perp|^2\beta^2}{(n\bar{n})}\partial_\omega^2\right]\left[\frac{\ln(\omega - i0)}{2(\omega-1)} - \frac{z_1\ln(\omega/z_1 - i0)}{\omega - z_1}\right.$$
$$\left. + \frac{z_1 \operatorname{Li}_2(1-\omega/z_1 + i0)}{\omega - z_1}\left(\frac{z_1}{\omega} + \frac{z_2}{\omega - 1}\right) - \frac{z_1\big(\operatorname{Li}_2(1) - \operatorname{Li}_2(z_2/z_1)\big)}{\omega - i0}\right]. \qquad (5.122)$$

For twist-4 the basic expression is

$$\Delta\mathcal{A}^{t=4} = \frac{-i}{4} \int \frac{d^4x\, e^{irx}}{\pi^2 x^2}(x, n + z_1\bar{n}) \int_0^1 \frac{du}{u^2}\left\{u^2(1 - u^2 + u^2\ln u)t\, z_1 z_2 \mathcal{O}_{A,+}(uz_1, uz_2)\right.$$
$$\left. + (1 + P_{12})\left[(1-u^2)\left(z_1\partial_{z_1} - \frac{z_2}{z_{21}}\right) + (1 - u^2 + u^2\ln u) z_1 \partial_{z_1}^2 z_{21}\right]\mathcal{R}_A(uz_1, uz_2)\right\}, \qquad (5.123)$$

where $\mathcal{R}_A$ is the obvious analogue of $\mathcal{R}_V$ introduced before. Basically the calculation can be performed using the same techniques we already encountered for the vector operator. From $\tilde{\Phi}_-$ one obtains

$$\Delta\mathcal{A}^{t=4}[\tilde{\Phi}_-] = \frac{\tilde{s}}{m}\int d\tilde{\Phi}_- \left\{\left[\frac{|P_\perp|^2\beta^2}{(n\bar{n})}\partial_\omega + \frac{t}{(n\bar{n})}\left(\partial_\omega\, \omega(\omega-1) - \frac{\beta}{\xi}\right)\right] \times\right.$$
$$\times \left[\frac{1}{2}\frac{\operatorname{Li}_2(\omega - i0) - \operatorname{Li}_2(1)}{\omega - 1} + \frac{z_1 \operatorname{Li}_2(1 - \omega/z_1 + i0)}{\omega - z_1}\left(1 - \frac{z_1}{\omega} - \frac{z_2}{\omega - 1}\right)\right.$$
$$\left. + \frac{3}{2}\frac{z_1\ln(\omega/z_1 - i0)}{\omega - z_1} - \frac{z_1\big(\operatorname{Li}_2(1) + \operatorname{Li}_2(z_2/z_1)\big)}{\omega - i0} - \frac{3}{4}\frac{\ln(\omega - i0)}{\omega - 1}\right]$$
$$\left. + \frac{t}{2(n\bar{n})}\partial_\omega\, z_1 z_2\left[2\frac{z_1\operatorname{Li}_2(1-\omega/z_1 + i0)}{\omega - z_1} + 3\frac{z_1 \ln z_1 - \omega\ln(\omega - i0)}{\omega - z_1}\right]\right\} \qquad (5.124)$$

and from $\tilde{h}_+$

$$\begin{aligned}\Delta\mathcal{A}^{t=4}[\tilde{h}_+] = \int d\tilde{h}_+ \bigg\{ \bigg[\bigg(\frac{(\tilde{v}\Delta)}{(n\bar{n})} + \frac{(\tilde{v}n)t}{(n\bar{n})^2} \bigg) \bigg(1 + \frac{\beta}{\xi}\partial_\omega \bigg) - \frac{(\tilde{v}n)|P_\perp|^2\beta^2}{2(n\bar{n})^2}\partial_\omega^2 \bigg] \times \\ \times \bigg[\frac{\mathrm{Li}_2(\omega - i0) - \mathrm{Li}_2(1)}{\omega - 1} - \frac{3}{2}\frac{\ln(\omega - i0)}{\omega - 1} + \frac{3\ln(\omega/z_1 - i0)}{\omega - z_1} \\ + \frac{2z_1\big(\mathrm{Li}_2(1) - \mathrm{Li}_2(z_2/z_1)\big)}{\omega - i0} + \frac{2z_1 \,\mathrm{Li}_2(1 - \omega/z_1 + i0)}{\omega - z_1} \bigg] \\ + \frac{(\tilde{v}n)t}{(n\bar{n})^2}\partial_\omega^2 \bigg[\frac{3}{4}(\omega + 2z_1z_2)\ln(\omega - i0) - \omega\,\mathrm{Li}_2(\omega + i0) \\ - z_1\ln(\omega/z_1 - i0)\bigg(\frac{3}{2}(\omega + z_2) - \frac{1}{2}\frac{z_1z_2}{\omega - z_1} \bigg) \\ + z_1\,\mathrm{Li}_2(1 - \omega/z_1 + i0)\bigg(\omega + z_2 + \frac{2z_1z_2}{\omega - z_1} \bigg) \bigg] \bigg\} . \qquad (5.125)\end{aligned}$$

Translation invariance is recovered in the sum

$$\Delta\mathcal{A} = \Delta\mathcal{A}^{t=2} + \Delta\mathcal{A}^{t=3} + \Delta\mathcal{A}^{t=4} . \qquad (5.126)$$

In order to outline how this works, let us focus on Φ_-. It is convenient to rewrite the answer for twist-2 as follows:

$$\begin{aligned}\Delta\mathcal{A}^{t=2}[\tilde{\Phi}_-] = \frac{\tilde{s}}{m}\int d\tilde{\Phi}_- \bigg\{ \ln(\omega - i0) - \frac{1}{2}\bigg[\frac{|P_\perp|^2\beta^2}{(n\bar{n})}\partial_\omega + \frac{t}{(n\bar{n})}\bigg(\omega - z_1 - \frac{\beta}{\xi} \bigg) \bigg] \times \\ \times \bigg[\frac{2z_1\,\mathrm{Li}_2(1 - \omega/z_1 + i0)}{\omega - z_1} + \frac{z_1\ln(\omega/z_1 - i0)}{w - z_1} \bigg] \bigg\} . \qquad (5.127)\end{aligned}$$

As one can check explicitly, the terms proportional to $\mathrm{Li}_2(1-\omega/z_1+i0)$, $\mathrm{Li}_2(z_2/z_1)$, $z_1\,\mathrm{Li}_2(1)$, $\ln(z_1)$ cancel completely in the sum. The remaining ones are independent of z_1 under the condition $z_2 = z_1 - 1$ and the answer can be cast into the form

$$\begin{aligned}\Delta\mathcal{A}[\tilde{\Phi}_-] = \frac{\tilde{s}}{m}\int d\tilde{\Phi}_- \bigg\{ \ln(\omega - i0) - \frac{1}{2}\bigg[\frac{|P_\perp|^2\beta^2}{(n\bar{n})}\partial_\omega + \frac{t}{(n\bar{n})}\bigg(\omega - 1 - \frac{\beta}{\xi} \bigg) \bigg] \times \\ \times \bigg[\frac{1}{2}\frac{\ln(\omega - i0)}{\omega - 1} - \frac{\mathrm{Li}_2(\omega + i0) - \mathrm{Li}_2(1)}{\omega - 1} \bigg] \bigg\} . \qquad (5.128)\end{aligned}$$

In the case of $\tilde{h}_+$ the same is true:

$$\begin{aligned}\Delta\mathcal{A}[\tilde{h}_+] = -\int d\tilde{h}_+ \bigg\{ \frac{(\tilde{v}n)}{(n\bar{n})}\frac{1}{\omega - i0} - \bigg[\frac{(\tilde{v}n)|P_\perp|^2\beta^2}{2(n\bar{n})^2}\partial_\omega^2 - \frac{(\tilde{v}\Delta)}{(n\bar{n})}\bigg(1 + \frac{\beta}{2\xi}\partial_\omega \bigg) \\ + \frac{(\tilde{v}n)t}{(n\bar{n})^2}\bigg(\frac{1}{2}\partial_\omega(\omega - 1) - 1 - \frac{\beta}{\xi}\partial_\omega \bigg) \bigg] \bigg[\frac{1}{2}\frac{\log(\omega - i0)}{\omega - 1} - \frac{\mathrm{Li}_2(\omega + i0) - \mathrm{Li}_2(1)}{\omega - 1} \bigg] \bigg\} . \qquad (5.129)\end{aligned}$$

This result is again notably simply and very similar to what was obtained in the parity-even calculation.

6. Discussion of the results

6.1. GPD expressions

The equations for the amplitudes $\mathcal{A}_{\mp\pm}$, $\mathcal{A}_{0\pm}$, $\mathcal{A}$ and $\Delta\mathcal{A}$ given in Eqs. (5.36), (5.47), (5.115), (5.116), (5.128) and (5.129) are one of the main new results of this work. They are still written in terms of double distributions and a formulation which uses the generalized parton distributions directly is highly desirable. This is possible with the help of the following set of formulas (for $n = 0, 1, 2$)

$$\begin{aligned}
\int d\Phi_+\, \beta^n \partial_\omega^{n-1} f(\omega) &= -(-2)^{n-1}\xi^{n+1}\partial_\xi^n \xi^{n-2}\, C_f \otimes E^+ \,, \\
\int d\tilde{\Phi}_-\, \beta^n \partial_\omega^{n-1} f(\omega) &= -(-2)^{n-1}\xi^{n+1}\partial_\xi^n \xi^{n-1}\, C_f \otimes \widetilde{E}^+ \,, \\
\int dh_-\, (\beta\partial_\omega)^n f(\omega) &= (-2\xi^2\partial_\xi)^n\, C_f \otimes M^+ \,, \\
\int d\tilde{h}_+\, (\beta\partial_\omega)^n f(\omega) &= (-2\xi^2\partial_\xi)^n\, C_f \otimes \widetilde{H}^+ \,,
\end{aligned} \tag{6.1}$$

where f is an arbitrary function and C_f will be defined below. Further, $M = H + E$ and the superscript "+" denotes the charge-even linear combination of a generic GPD F

$$F^+(x,\xi,t) = F(x,\xi,t) - \sigma(F) F(-x,\xi,t)\,, \tag{6.2}$$

with

$$\sigma(F) = \begin{cases} +1 & \text{for } F = H, E, \\ -1 & \text{for } F = \widetilde{H}, \widetilde{E}. \end{cases} \tag{6.3}$$

The symbol "$C_f \otimes$" represents a shorthand notation for the convolution over the momentum fraction x,

$$C_f \otimes F \equiv \int_{-1}^{1} dx\, C_f(x,\xi) F(x,\xi,t)\,, \tag{6.4}$$

and in Eq. (6.1)

$$C_f(x,\xi) = f\left(\frac{x+\xi}{2\xi}\right). \tag{6.5}$$

A proof for the collection of reduction formulas (6.1) is supplemented in App. C.

We take the opportunity to use $a_{\mp\pm} = 2\sqrt{(n\bar{n})}(\varepsilon^\pm a)$ for an arbitrary four-vector a, as well as $(n\bar{n}) = (Q^2 + t)/2 = -(qq')$ and with the help of (6.1) we find at our often-quoted accuracy

$$
\begin{aligned}
\mathcal{A}_{\mp\pm} = -\frac{8(\epsilon^{\pm}P)}{Q^2}\bigg\{ & \left[(\epsilon^{\pm}v) - \frac{2(vn)(\epsilon^{\pm}P)}{Q^2}\xi^2\partial_\xi\right]\xi^2\partial_\xi\,(xC_1)\otimes M^+ \\
& \pm\left[(\epsilon^{\pm}\tilde{v}) - \frac{2(\tilde{v}n)(\epsilon^{\pm}P)}{Q^2}\xi^2\partial_\xi\right]\xi^2\partial_\xi\,\xi\,C_1\otimes\widetilde{H}^+ \\
& + \frac{(vP)(\epsilon^{\pm}P)}{2m^2}\xi^3\partial_\xi^2\,(xC_1)\otimes E^+ \mp \frac{(\tilde{v}\Delta)(\epsilon^{\pm}P)}{4m^2}\xi^3\partial_\xi^2\,\xi^2\,C_1\otimes\widetilde{E}^+\bigg\},
\end{aligned}
\tag{6.6}
$$

$$
\begin{aligned}
\mathcal{A}_{0\pm} = \frac{2}{Q}\bigg\{ & \left[(\epsilon^{\pm}v) - 4\frac{(vn)(\epsilon^{\pm}P)}{Q^2}\xi^2\partial_\xi\right]\xi\,C_1\otimes M^+ + \frac{(vP)(\epsilon^{\pm}P)}{m^2}\xi^2\partial_\xi\,C_1\otimes E^+ \\
& \pm\left[(\epsilon^{\pm}\tilde{v}) - 4\frac{(\tilde{v}n)(\epsilon^{\pm}P)}{Q^2}\xi^2\partial_\xi\right]\xi\,C_1\otimes\widetilde{H}^+ \pm \frac{(\tilde{v}\Delta)(\epsilon^{\pm}P)}{2m^2}\xi^2\partial_\xi\,\xi\,C_1\otimes\widetilde{E}^+\bigg\},
\end{aligned}
\tag{6.7}
$$

and

$$
\begin{aligned}
\mathcal{A} &= \frac{1}{2}\big(\mathcal{A}_{++} + \mathcal{A}_{--}\big) = \frac{(vP)}{2m^2}\,\mathbb{V}_1 + \frac{(vq')}{(qq')}\,\mathbb{V}_2\,, \\
\Delta\mathcal{A} &= \frac{1}{2}\big(\mathcal{A}_{++} - \mathcal{A}_{--}\big) = \frac{(\tilde{v}\Delta)}{4m^2}\,\widetilde{\mathbb{V}}_1 + \frac{(\tilde{v}q')}{(qq')}\,\widetilde{\mathbb{V}}_2\,,
\end{aligned}
\tag{6.8}
$$

with

$$
\begin{aligned}
\mathbb{V}_1 &= \left(1 - \frac{t}{2Q^2}\right)C_0\otimes E^+ + \frac{t}{Q^2}C_1\otimes E^+ - \frac{2}{Q^2}\left(\frac{t}{\xi} + 2|P_\perp|^2\xi^2\partial_\xi\right)\xi^2\partial_\xi\,C_2\otimes E^+ \\
&\quad + \frac{8m^2}{Q^2}\xi^2\partial_\xi\xi\,C_2\otimes M^+, \\
\mathbb{V}_2 &= \left(1 - \frac{t}{2Q^2}\right)\xi\,C_0\otimes M^+ + \frac{t\,\xi}{Q^2}C_1\otimes M^+ - \frac{4}{Q^2}\left(\frac{t}{\xi} + |P_\perp|^2\xi^2\partial_\xi\right)\xi^2\partial_\xi\,\xi\,C_2\otimes M^+ \\
&\quad + \frac{2t}{Q^2}\,\xi\,C_2\otimes M^+,
\end{aligned}
\tag{6.9}
$$

and

$$
\begin{aligned}
\widetilde{\mathbb{V}}_1 &= \left(1 - \frac{t}{2Q^2}\right)\xi\,C_0\otimes\widetilde{E}^+ + \frac{t\,\xi}{Q^2}C_1\otimes\widetilde{E}^+ - \frac{2}{Q^2}\left(\frac{t}{\xi} + 2|P_\perp|^2\xi^2\partial_\xi\right)\xi^2\partial_\xi\,\xi\,C_2\otimes\widetilde{E}^+ \\
&\quad + \frac{8m^2}{Q^2}\xi^2\partial_\xi\,C_2\otimes\widetilde{H}^+\,, \\
\widetilde{\mathbb{V}}_2 &= \left(1 - \frac{t}{2Q^2}\right)\xi\,C_0\otimes\widetilde{H}^+ + \frac{t\,\xi}{Q^2}C_1\otimes\widetilde{H}^+ - \frac{4}{Q^2}\left(\frac{t}{\xi} + |P_\perp|^2\xi^2\partial_\xi\right)\xi^2\partial_\xi\,\xi\,C_2\otimes\widetilde{H}^+ \\
&\quad + \frac{2t}{Q^2}\,\xi\,C_2\otimes\widetilde{H}^+\,,
\end{aligned}
\tag{6.10}
$$

where the coefficient functions are given by

$$\begin{aligned} C_0(x,\xi) &= \frac{1}{x+\xi-i0}\,, \\ C_1(x,\xi) &= \frac{\ln\!\left(\frac{\xi+x}{2\xi}-i0\right)}{x-\xi}\,, \\ C_2(x,\xi) &= \frac{\mathrm{Li}_2\!\left(\frac{\xi-x}{2\xi}+i0\right)-\mathrm{Li}_2(1)}{x+\xi}+\frac{1}{2}C_1(x,\xi)\,. \end{aligned} \tag{6.11}$$

The scope of the derivative $\partial_\xi = \partial/\partial\xi$ is such that it acts on *everything* that stands to the right of it, i.e. on the coefficient functions and the GPDs.

6.2. Analyticity

6.2.1. On factorization

There is still an open issue that we have not yet addressed: is the answer for the helicity amplitudes well-defined? To answer this question one has to examine the analytic structure of the coefficient functions that we have encountered. For example, the coefficient function C_0, see Eq. (6.11), has a simple pole at $x+\xi=0$, which is regulated by the $i0$ prescription. C_0 can be written in a distributional sense

$$C_0(x,\xi) = i\pi\delta(x+\xi) + \mathrm{P.V.}\,\frac{1}{x+\xi}\,, \tag{6.12}$$

where P. V. stands for the Cauchy principal value. It follows that the convolution $C_0 \otimes F^+$ with a generic GPD $F^+ \in \{H^+, E^+, \widetilde{H}^+, \widetilde{E}^+\}$ is well-defined (finite) for $\xi > 0$. At leading order (LO), the imaginary parts of the functions $\mathbb{V}_{1,2}$, $\widetilde{\mathbb{V}}_{1,2}$ are simply given by the GPDs on the cross-over line, for example

$$\mathrm{Im}\,\mathbb{V}_1 \overset{\mathrm{LO}}{=} -\pi\big(E(\xi,\xi,t) - E(-\xi,\xi,t)\big)\,. \tag{6.13}$$

Derivatives have to be handled with special caution: although the GPDs are finite and continuous on the DGLAP-ERBL boundaries ($x=\pm\xi$), their derivatives in general are not. For example the following expression

$$\mathrm{Im}\int_{-1}^{1} dx\,\frac{1}{(\xi-x-i0)^2}F(x,\xi,t) = \pi(\partial_x F)(\xi,\xi) \tag{6.14}$$

would be ill-defined. A perturbative analysis [22] indicates that $(\partial_x F)(x,\xi)$ has a "jump" at $x=\xi$, i.e. $(\partial_x F)(\xi+0,\xi) \neq (\partial_x F)(\xi-0,\xi)$, which is also compatible with the evolution equations for GPDs. In general it cannot be excluded that $(\partial_x F)(\xi,\xi)$ may be singular, the precise or even the asymptotic form is not known (to the best of the author's knowledge). The same applies also to $(\partial_\xi F)(\xi,\xi)$ as well as the other crossover point ($x=-\xi$). Terms of the form (6.14) would put the validity of our approach into question and/or indicate a breakdown of factorization. We are going to show that this does not happen.

To do so, we only need to examine $C_{1,2}$, or equivalently C_1 and

$$C_{\mathrm{Li}}(x,\xi) = \frac{\mathrm{Li}_2\left(\frac{\xi-x}{2\xi} + i0\right) - \mathrm{Li}_2(1)}{x+\xi}\,. \tag{6.15}$$

Splitting the relevant expressions in imaginary and real parts, one gets for $\xi > 0$

$$\begin{aligned} C_1(x,\xi) &= -i\frac{\pi\theta(-\xi-x)}{x-\xi} + \frac{\ln\left|\frac{\xi+x}{2\xi}\right|}{x-\xi}\,, \\ C_{\mathrm{Li}}(x,\xi) &= i\frac{\pi\theta(-x-\xi)\ln\left(\frac{\xi-x}{2\xi}\right)}{\xi+x} + \frac{\mathrm{Li}_2^{\mathrm{abs}}\left(\frac{\xi-x}{2\xi}\right) - \mathrm{Li}_2(1)}{x+\xi}\,, \end{aligned} \tag{6.16}$$

where

$$\mathrm{Li}_2^{\mathrm{abs}}(z) = -\int_0^z \frac{dy}{y}\ln|1-y|\,, \tag{6.17}$$

which is always real for real z and coincides with $\mathrm{Li}_2(z)$ for all $z \le 1$. We see that $C_1(x,\xi)$ has a logarithmic singularity at $x = -\xi$, while it is regular for $x = \xi$. The same is true for $C_{\mathrm{Li}}(x,\xi)$, where its logarithmic singularity can be seen by using the representation

$$C_{\mathrm{Li}}(x,\xi) = (2\xi)^{-1}\frac{1}{z}\int_{1-z+i0}^{1}\frac{dy}{y}\,\ln(1-y)\,, \quad \text{with } z = \frac{x+\xi}{2\xi}\,, \tag{6.18}$$

from which we obtain the behavior of $C_{\mathrm{Li}}(x,\xi)$ in the vicinity of $x \sim -\xi$ (or $z \sim 0$):

$$C_{\mathrm{Li}}(x,\xi) \overset{x\to-\xi}{\longrightarrow} \frac{\ln\left(\frac{\xi+x}{2\xi} - i0\right)}{2\xi}\,. \tag{6.19}$$

Since logarithmic singularities are integrable, we conclude that $C_{1,2} \otimes F^+$ is perfectly well-defined and we only need to check that derivatives with respect to ξ do not pose any problems.

To make progress let us fix some generic notation: let $C(x,\xi)$ and $F(x,\xi)$ be functions with the same analytic properties as the coefficient functions $C_{1,2}$ and the GPDs respectively. The skewness ξ is assumed to lie in the range $0 < \xi < 1$. $C(x,\xi)$ can be written as $\xi^{-1}c\left(\frac{x}{\xi}\right)$ with some function c. Then we consider the derivative operator $\xi\partial_\xi$ acting on the convolution

$$\xi\partial_\xi\, C\otimes F = \frac{1}{\xi}\int_{-1}^{1}\!dx\left[c\left(\frac{x}{\xi}\right)\xi\partial_\xi F(x,\xi) - F(x,\xi)x\partial_x c\left(\frac{x}{\xi}\right)\right] + \dots\,, \tag{6.20}$$

where, here and below, the ellipses stand for "unproblematic" contributions (here: the term produced by differentiating ξ^{-1}, which yields again a regular contribution as we argued above). Integration by parts in the second term yields

$$\xi\partial_\xi\, C\otimes F = \int_{-1}^{1}\!dx\, C(x,\xi)(\xi\partial_\xi + x\partial_x)F(x,\xi) + \dots\,, \tag{6.21}$$

where yet another unproblematic term was absorbed into the ellipses. If $(\xi\partial_\xi + x\partial_x)F(x,\xi)$ is bounded (e.g. when $F(\xi,\xi)$ is continuous, which would imply that $\partial_\xi F(\xi,\xi)$ is bounded), then the integral (6.21) converges. In the case of a possible singularity of the derivatives, we note that the differential operator $\xi\partial_\xi + x\partial_x$ does not enhance the singularity. To make this explicit one can define variables $\zeta_\pm = x \pm \xi$ to find $\xi\partial_\xi + x\partial_x = \zeta_+\partial_{\zeta_+} + \zeta_-\partial_{\zeta_-}$. Suppose that the nonanalytic behavior of a GPD F is logarithmic, say $(\partial_x F)(x,\xi) \overset{x\to\pm\xi}{\sim} \ln(x \mp \xi)$, this does not endanger the convergence of (6.21). Even $(\partial_x F)(x,\xi) \overset{x\to\pm\xi}{\sim} (x \mp \xi)^{-\alpha}$ with $\alpha < 1$ would still result in a convergent integral.

The well-definedness of the second and in fact all higher derivatives follows from repetition of the argument above. Thus all convolutions define smooth functions of ξ for $\xi > 0$. Note that the same line of reasoning is extended trivially to the convolution with xC_1, as appearing in the double-flip amplitudes $\mathcal{A}_{\mp\pm}$, see Eq. (6.6).

6.2.2. Dispersion relations

In the absence of a D-term contribution, the convolution of a GPD with C_0 satisfies a so-called (unsubtracted) *dispersion relation*, e.g. in the $\sigma = +1$ case

$$(\mathrm{Re}\, C_0 \otimes M^+)(\xi) = \mathrm{P.V.}\, \frac{1}{\pi} \int_0^1 dx\, \frac{2x}{\xi^2 - x^2} (\mathrm{Im}\, C_0 \otimes M^+)(x)\,, \tag{6.22}$$

or in the $\sigma = -1$ sector

$$(\mathrm{Re}\, C_0 \otimes \widetilde{H}^+)(\xi) = \mathrm{P.V.}\, \frac{1}{\pi} \int_0^1 dx\, \frac{2\xi}{\xi^2 - x^2} (\mathrm{Im}\, C_0 \otimes \widetilde{H}^+)(x)\,. \tag{6.23}$$

We will now convince ourselves that such a property also holds for the DVCS power corrections. Before going into details, let us make a couple of simplifying assumptions. First we assume that we are dealing with a GPD $\widetilde{F}$, which is symmetric in x, i.e. $\widetilde{F}(x,\xi) = \widetilde{F}(-x,\xi)$. Next, we assume that $\widetilde{F}$ is "D-term/pion-pole free", such that $\widetilde{F}$ can be represented by a single DD $\tilde{f}$,

$$\widetilde{F}(x,\xi) = \int d\beta\, d\alpha\, \delta(x - \beta - \alpha\xi) \tilde{f}(\beta,\alpha)\,. \tag{6.24}$$

The above assumptions would be true e.g. for $\widetilde{H}^+$. The following can also be extended easily for antisymmetric GPDs like M^+ without problems. Initially one notices that the imaginary part of the kernels C_i has its support only in the "outer" regions $|x| \geq \xi$, see Eqs. (6.12) and (6.16) and can be written as $\mathrm{Im}\, C_i(x,\xi) = \theta(x-\xi)\frac{1}{x} c_i(\xi/x)$ without loss of generality. It requires just a shift $x \to -x$ in the convolution. Starting from an expression of the type "r.h.s. of (6.23)" one can bring it into the form

$$\mathrm{P.V.} \int_0^1 dx\, \frac{2\xi}{\xi^2 - x^2} (\mathrm{Im}\, C_i \otimes \widetilde{F})(x) = \mathrm{P.V.} \int_{-1}^1 \frac{dx}{\xi - x} \int_{|x|}^1 \frac{dy}{y}\, c_i(y) \widetilde{F}(x/y, x) \tag{6.25}$$

by using the symmetry of $\widetilde{F}$. Inserting the DD parametrization (6.24) we get for the r.h.s. of the above equation

$$\int d\beta\, d\alpha\, \tilde{f}(\beta,\alpha) \int_0^1 \frac{dy}{y}\, c_i(y)\, \mathrm{P.V.} \int_{-y}^{y} \frac{dx}{\xi - x}\, \delta\big(x(1-\alpha y) - y\beta\big)\,. \tag{6.26}$$

We can now integrate over the δ-function and attach the P. V. prescription to the y-integral. The root of the argument of the δ-function is always inside the interval $[-y, y]$: the range of integration for β, α is such that $|\beta| + |\alpha| \leq 1$, therefore obviously $|\beta| + |\alpha y| \leq 1$ and thus $|y\beta|/|1 - \alpha y| \leq |y|$, which is exactly the condition for the x-integral to be nonzero. Then (6.26) is equal to

$$\int d\beta\, d\alpha\, \tilde{f}(\beta, \alpha)\, \mathrm{P.V.} \int_0^1 dy\, \frac{c_i(y)}{(1-\alpha y)\xi - \beta y}\,. \tag{6.27}$$

In principle, by using

$$\mathrm{P.V.}\,\frac{1}{\zeta} = \mathrm{Re}\,\frac{1}{\zeta - i\epsilon}\,, \quad \epsilon \to 0\,, \tag{6.28}$$

one could proceed with explicit expressions for c_i with $i = 0, 1, 2$ to obtain the desired dispersion relation

$$\mathrm{Re}\, C_i \otimes \widetilde{F} = \mathrm{P.V.} \int_0^1 dx\, \frac{2\xi}{\xi^2 - x^2} (\mathrm{Im}\, C_i \otimes \widetilde{F})(x) \tag{6.29}$$

and analogously for a GPD F antisymmetric in x

$$\mathrm{Re}\, C_i \otimes F = \mathrm{P.V.} \int_0^1 dx\, \frac{2x}{\xi^2 - x^2} (\mathrm{Im}\, C_i \otimes F)(x)\,. \tag{6.30}$$

These formulas also follow from a common property of the C_i: Let $K_i(x, \xi)$ be defined as $C_i(-x, \xi)$ *without* the $i0$ shift. Then

$$\mathrm{Im}\, K_i(x, \xi) = \frac{1}{2i}\big(K_i(x + i\eta, \xi) - K_i(x - i\eta, \xi)\big)\,. \tag{6.31}$$

with a positive infinitesimal parameter $\eta \to 0$. Generically $K_i(x, \xi)$ has nonanalytic behavior on the real axis for $x \geq \xi$. Turning back to (6.27) and letting $y = 1/(2z - 1)$ and ω as in Eq. (5.14), one can write

$$\mathrm{P.V.} \int_0^1 dy\, \frac{c_i(y)}{(1-\alpha y)\xi - \beta y} = \frac{1}{\xi}\, \mathrm{Re} \int_1^\infty \frac{dz}{2z - 1}\, \frac{c_i\left(\frac{1}{2z-1}\right)}{z - \omega - i\epsilon}\,. \tag{6.32}$$

Reinserting $c_i(u) = \xi/u\ \mathrm{Im}\, K(\xi/u, \xi)$ into the above equation we find

$$(6.32) = \mathrm{Re}\, \frac{1}{2i} \int_1^\infty dz\, \frac{K_i\big(\xi(2z-1) + i\eta, \xi\big) - K_i\big(\xi(2z-1) - i\eta, \xi\big)}{z - \omega - i\epsilon}\,. \tag{6.33}$$

This integral can be evaluated using Cauchy's theorem, schematically depicted in Fig. 6.1. The original integrations run infinitesimally shifted from the axis where the K_i are nonanalytic, depicted by the red cross with the attached red line. By closing the original paths with the arcs in Fig. 6.1, which have vanishing contributions, we only pick up the residue of the pole at $z = \omega + i0$ (upper red cross). Finally

$$(6.33) = \mathrm{Re}\, \pi K_i\big(\xi(2\omega - 1) + i0, \xi\big)\,, \tag{6.34}$$

and by plugging this result into Eq. (6.27) and using Eq. (6.1) proves the claimed dispersion relations. Note that for the contribution of the arcs at infinity to vanish it is necessary that $c_i(x, \xi)$ vanish for $|x| \to \infty$. This is true for the convolutions with $C_{0,1,2}$, but not for those

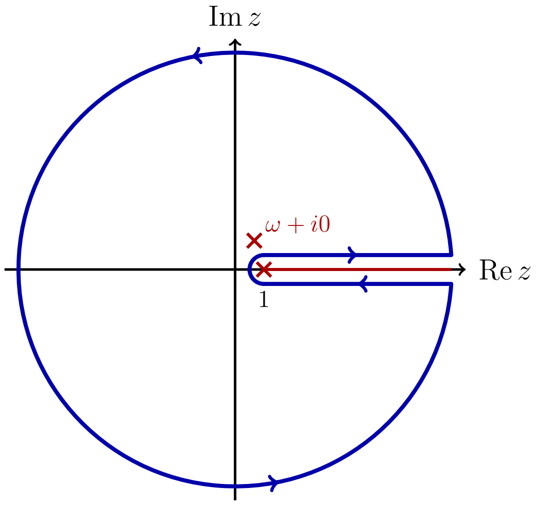

Figure 6.1.: Closed contour integral for the evaluation of (6.33).

of the type $(xC_1)\otimes\ldots$. Here the arcs yield a constant term, which disappears in the answer for the amplitudes, because only the ξ-derivative of $(xC_1)\otimes\ldots$ contributes.

Note that the GPDs H and E have a D-term [43], in this case the dispersion relation (6.30) needs to be completed by a subtraction constant on the r.h.s. Eq. (6.30) holds for $M = H+E$, since the D-terms for H and E are the same except for an opposite sign, which makes M D-term free. One can generically quote the D-term as

$$D(x,\xi) = \theta(\xi^2 - x^2)\,\mathrm{sgn}(\xi)\varphi_D\left(\frac{x}{\xi}\right) \tag{6.35}$$

for a fixed flavor. The t- and μ^2-dependence is left implicit and φ_D is an odd function of its argument. Since $D(x,\xi)$ is essentially an antisymmetric function of the ratio x/ξ, that vanishes when $x \to \xi$, one gets for $\xi > 0$

$$\begin{aligned}
C_0\otimes D &= -\int_{-1}^{1} \frac{dy}{1-y}\,\varphi_D(y)\,, \\
C_1\otimes D &= -\int_{-1}^{1} \frac{dy}{1-y}\,\ln\left(\frac{1+y}{2}\right)\varphi_D(y)\,, \\
(xC_1)\otimes D &= -\xi\int_{-1}^{1} \frac{dy}{1-y}\,y\ln\left(\frac{1+y}{2}\right)\varphi_D(y)\,, \\
C_2\otimes D &= -\int_{-1}^{1} \frac{dy}{1-y}\left[\mathrm{Li}_2\left(\frac{1+y}{2}\right) - \mathrm{Li}_2(1) + \frac{1}{2}\ln\left(\frac{1+y}{2}\right)\right]\varphi_D(y)\,.
\end{aligned} \tag{6.36}$$

As a consequence of this simple ξ-dependence, the only place where the D-term really contributes is to the real part of $\mathbb{V}_1$, see Eq. (6.10).

A similar observation concerns the pion-pole contribution, which accompanies the GPD

$\widetilde{E}$ and is typically written as

$$\widetilde{E}_\pi = \theta(\xi^2 - x^2)\,\mathrm{sgn}(\xi)\frac{1}{\xi}\varphi_\pi\left(\frac{x}{\xi}\right), \tag{6.37}$$

where φ_π is an even function. Then the set of formulas analogous to (6.36) reads

$$\begin{aligned}
C_0\otimes\widetilde{E}_\pi &= \frac{1}{\xi}\int_{-1}^{1}\frac{dy}{1-y}\,\varphi_\pi(y)\,,\\
C_1\otimes\widetilde{E}_\pi &= -\frac{1}{\xi}\int_{-1}^{1}\frac{dy}{1-y}\,\ln\left(\frac{1+y}{2}\right)\varphi_\pi(y)\,,\\
C_2\otimes\widetilde{E}_\pi &= \frac{1}{\xi}\int_{-1}^{1}\frac{dy}{1-y}\left[\mathrm{Li}_2\left(\frac{1+y}{2}\right) - \mathrm{Li}_2(1) - \frac{1}{2}\ln\left(\frac{1+y}{2}\right)\right]\varphi_\pi(y)\,.
\end{aligned} \tag{6.38}$$

It is then easy to see that the pion-pole only contributes to the amplitude $\widetilde{\mathbb{V}}_1$.

Often one uses an expansion in terms of Gegenbauer polynomials with index 3/2, e.g. $\varphi_{D,\pi}(y) \sim (1-y^2)\sum_j a_j C_j^{\frac{3}{2}}(y)$, which are eigenfunctions of the evolution equations. In that case one can calculate the integrals in (6.36) and (6.38) analytically. We will supplement details at the end of Sec. 6.5.

6.3. A byproduct: pion DVCS

For the sake of completeness, let us briefly mention that the answer for the helicity amplitudes including the power corrections $\sim t/Q^2, m/Q^2$ for DVCS off a scalar target (pion) is formally contained in what we have obtained so far. Due to technical advantages, the results for this process were calculated first and reported in Ref. [42]. The nucleon case was addressed later in [44].

Pretend we want to repeat the calculation for a pion target. The first simplification is the observation that *pion* matrix elements of the axial operator $\mathcal{O}_A$ vanish identically. Secondly, out of the matrix element of the vector operator $\mathcal{O}_V$ only the isoscalar part survives the antisymmetrization in the field positions z_1, z_2, see [42]. Being inherently antisymmetric under $z_1 \leftrightarrow z_2$, the isoscalar part is unaffected by this operation. That means for this purpose one can take, cf. [42]

$$\mathcal{O}^\pi_{++}(z_1,z_2) = \frac{5}{18}e^2\left[\bar{u}(z_1 n)\not{n}u(z_2 n) + \bar{d}(z_1 n)\not{n}d(z_2 n)\right]. \tag{6.39}$$

The (isoscalar) pion GPD is defined by

$$\langle\pi^b(p')|\,\mathcal{O}^\pi_{++}(z_1,z_2)\,|\pi^a(p)\rangle = 2(Pn)\delta^{ab}\varkappa\int dx\, e^{-i(Pn)[z_1(\xi-x)+z_2(\xi+x)]}H^\pi(x,\xi,t)\,, \tag{6.40}$$

where $\varkappa = \frac{5}{18}e^2$ and a, b are isospin indices. The corresponding DD reads

$$\langle\pi^b(p')|\,\mathcal{O}^\pi_{++}(z_1,z_2)\,|\pi^a(p)\rangle = \delta^{ab}\varkappa\int d\beta\,d\alpha\, e^{-i\ell_{12}n}\left[2(Pn)f(\beta,\alpha,t) - (\Delta n)g(\beta,\alpha,t)\right], \tag{6.41}$$

which can be brought into the form of Eq. (5.2) without the (vn), $(\tilde{v}n)$ and $\tilde{s}$ terms and with $s \to 2m\varkappa$. Consequently it is obvious, that in the pion-sector, the helicity amplitudes in DD representation are entirely given by (5.36), (5.47), (5.116), and (5.128), keeping only the pion-analogue of Φ_+ and $s \to 2m\varkappa$.

Similarly, the GPD expression can be obtained, either directly using the results from App. C, or from Eqs. (6.6), (6.7), (6.8), (6.9), with $(M^+, \widetilde{H}^+, \widetilde{E}^+) \to 0$, $E^+ \to H^\pi$ and $(vP) \to 2m^2\varkappa$. Note that $\mathcal{A}^{++} = \mathcal{A}^{--}$ which is also a consequence of parity conservation. All statements of Sec. 6.2 remain valid without modification. Further details and model estimates can be found in [42]. We will however not bother ourselves with scalar DVCS any further and focus on the phenomenologically more relevant nucleon scattering.

6.4. Comparison with existing results

6.4.1. On the relation to Ref. [15] and the large-Q^2 limit

Given that construction of the amplitude tensor and its calculation was basically "from scratch", it is mandatory to check whether the leading power behavior of $\mathcal{A}_{\mu\nu}$ agrees with existing expressions in the literature. If we focus only on the leading accuracy, $\mathcal{O}(Q^0)$, it is sufficient to keep the helicity conserving amplitudes, i.e.

$$\mathcal{A}_{\mu\nu} = \frac{1}{2}(\varepsilon^+_\mu \varepsilon^-_\nu + \varepsilon^-_\mu \varepsilon^+_\nu)(\mathcal{A}_{++} + \mathcal{A}_{--}) + \frac{1}{2}(\varepsilon^+_\mu \varepsilon^-_\nu - \varepsilon^-_\mu \varepsilon^+_\nu)(\mathcal{A}_{++} - \mathcal{A}_{--}) + \dots , \tag{6.42}$$

which is equivalent, see Eqs. (4.26), (4.28), to

$$\mathcal{A}_{\mu\nu} = -g^\perp_{\mu\nu}\,\mathcal{A} + i\varepsilon^\perp_{\mu\nu}\,\Delta\mathcal{A} + \dots , \tag{6.43}$$

where the ellipses stand for the power-suppressed helicity flip contributions. Performing a contraction with the polarization vectors, one gets

$$(\varepsilon^\mp|\mathcal{A}|\varepsilon^\pm) = \frac{(vP)}{2m^2} C_0 \otimes E^+ + \frac{(vq')}{(qq')}\xi\, C_0 \otimes (H^+ + E^+) \pm \frac{(\tilde{v}\Delta)}{4m^2}\,\xi C_0 \otimes \widetilde{E}^+ \pm \frac{(\tilde{v}q')}{(qq')}\xi\, C_0 \otimes \widetilde{H}^+ , \tag{6.44}$$

where $(\varepsilon^\mp|\mathcal{A}|\varepsilon^\pm) \equiv \varepsilon^\mp_\mu \mathcal{A}^{\mu\nu} \varepsilon^\pm_\nu$. To leading power accuracy, one can neglect the transverse components of P and thus

$$(\tilde{v}\Delta) \approx 2(vP) \approx -\frac{t}{\xi^2}\sqrt{1-\xi^2} , \tag{6.45}$$

where we dropped the term $\sim t$ in Eq. (5.18) and implicitly assumed proton polarization "$\uparrow$". In the same approximation we can set $t/m = -4\xi^2/(1-\xi^2)$ and we obtain

$$(\varepsilon^\mp|\mathcal{A}|\varepsilon^\pm) = -\sqrt{1-\xi^2} C_0 \otimes \big(H^+ \pm \widetilde{H}^+\big) + \frac{\xi^2}{\sqrt{1-\xi^2}} C_0 \otimes \big(E^+ \pm \widetilde{E}^+\big) . \tag{6.46}$$

This expression needs to be compared to Ref. [15]. In order to be consistent with [15] we need to adapt our choice of the light-cone, polarizations etc. to the reference frame chosen there. This requires $(n^\mu) \sim (1, 0, 0, -1)$ and $(\bar{n}^\mu) \sim (1, 0, 0, 1)$ up to a total normalization which is irrelevant. The amplitudes given in [15] correspond to a situation, where $(p^\mu) = (p^0, 0, 0, p^3 > 0)$ and $(p'^\mu) = (p'^0, p'^1 \le 0, 0, p'^3 > 0)$. Then one can construct according to our prescription $\varepsilon_\mu^\pm = \frac{1}{2|P_\perp|}(P_\mu^\perp \pm i\bar{P}_\mu^\perp)$, which yields

$$(\varepsilon_\mu^\pm) = \frac{(-1)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}, \tag{6.47}$$

and therefore, up to an overall (and unimportant) sign[1]

$$\epsilon_{[15]}(\pm) = \pm\varepsilon^\mp, \tag{6.48}$$

where the subscript denotes the notation in the corresponding reference. It follows that

$$M_{++,++,[15]} = (\varepsilon^+|\mathcal{A}|\varepsilon^-) \tag{6.49}$$

and from the explicit expressions

$$\begin{aligned} C_0 \otimes H^+ &= -\sum_q e_q^2 \int_{-1}^1 dx \left(\frac{1}{\xi - x - i0} - \frac{1}{\xi - x - i0} \right) H^q(x,\xi,t), \\ C_0 \otimes \widetilde{H}^+ &= +\sum_q e_q^2 \int_{-1}^1 dx \left(\frac{1}{\xi - x - i0} + \frac{1}{\xi - x - i0} \right) \widetilde{H}^q(x,\xi,t), \end{aligned} \tag{6.50}$$

one sees that our leading power result reproduces exactly the results reported in [15]. Note that in addition the same set of equations with $H \to E$, $\widetilde{H} \to \widetilde{E}$ hold.

6.4.2. On the relation to Ref. [40] – twist-3 and partial twist-4

Kivel and Mankiewicz presented results for the helicity flip amplitudes in [40]. We would like to compare them to our answer. To this end we have to quote a couple of formulas from [40].

In the following the subscript "[40]" on any quantity means that it was defined in [40]. If the quantity coincides with a definition of this work, or at least to a sufficient power accuracy, the label will be omitted.

The amplitude tensor

$$T_{[40]}^{\mu\nu} = -i \int d^4x \, e^{-i(q+q')x/2} \langle p'| \, j^\mu(x/2) j^\nu(-x/2) \, |p\rangle \tag{6.51}$$

is defined with a different sign compared to Eq. (4.33), i.e.

$$T_{[40]}^{\mu\nu} = -\mathcal{A}^{\mu\nu}. \tag{6.52}$$

The authors continue to define a longitudinal polarization vector $\varepsilon_{L[40]}^\mu$ for the virtual photon

$$\varepsilon_{L[40]}^\mu = \frac{1}{Q}\big(4\xi P^\mu + q^\mu\big). \tag{6.53}$$

[1]It is not clear in [15] if the expressions given for $\epsilon(\pm)$ correspond to covariant or contravariant vectors.

Out of the contraction $\varepsilon^{\mu}_{L[40]}T_{\mu\nu[40]}$ only the term $\sim P^\mu$ contributes due to the Ward identity:

$$\varepsilon^{\mu}_{L[40]}T_{\mu\nu[40]} = \frac{4\xi}{Q}P^\mu T_{\mu\nu[40]} \,. \tag{6.54}$$

If one expresses P in terms of our polarization vectors, one finds

$$P_\mu = (\varepsilon^0 P)\varepsilon^0_\mu - \frac{1}{4\xi}q_\mu - (\varepsilon^- P)\varepsilon^+_\mu - (\varepsilon^+ P)\varepsilon^-_\mu \,, \tag{6.55}$$

where $(\varepsilon^0 P) \approx Q/(4\xi)$. Equating (6.54) through (6.52) establishes a relation between amplitudes:

$$\varepsilon^{\mu}_{L[40]}T_{\mu\nu[40]} = -\big(\mathcal{A}_{0+}\varepsilon^-_\nu + \mathcal{A}_{0-}\varepsilon^+_\nu\big) - \frac{4\xi}{Q}\big((\varepsilon^+ P)\varepsilon^-_\nu\mathcal{A}_{++} + (\varepsilon^- P)\varepsilon^+_\nu\mathcal{A}_{--}\big)\,. \tag{6.56}$$

The remaining index ν is transverse and we can simply contract it with our $\varepsilon^\pm$, yielding

$$(\varepsilon_L|T|\varepsilon^\pm)_{[40]} = \mathcal{A}_{0\pm} + \frac{4\xi(\varepsilon^\pm P)}{Q}\mathcal{A}_{\pm\pm} \,, \tag{6.57}$$

where $(\varepsilon_L|T|\varepsilon^\pm)_{[40]}$ is a shorthand for $\varepsilon^{\mu}_{L[40]}T_{\mu\nu[40]}\varepsilon^{\pm,\nu}$. Explicitly, Kivel and Mankiewicz obtained for the l.h.s. of this equation

$$(\varepsilon_L|T|\varepsilon^\pm)_{[40]} = \frac{2\xi}{Q}\varepsilon^\pm_\nu g^{\nu\rho}_{\perp[40]}\int_{-1}^{1}\!dx\,\big[F_{\rho[40]}(x,\xi)C^+_{[40]}(x,\xi) - i\epsilon^\perp_{\rho\sigma[40]}\widetilde{F}^\sigma_{[40]}(x,\xi)C^-_{[40]}(x,\xi)\big]\,, \tag{6.58}$$

where the transverse tensors read in terms of the process momenta

$$g^{\nu\rho}_{\perp[40]} = g^{\nu\rho} - \frac{4\xi}{Q^2}\big(P^\nu q^\rho + q^\nu P^\rho + 4\xi P^\nu P^\rho\big)\,, \quad \epsilon^\perp_{\rho\sigma[40]} = \frac{4\xi}{Q^2}\varepsilon_{\rho\sigma\alpha\beta}q^\alpha P^\beta \tag{6.59}$$

and the coefficient functions are given by

$$C^\pm_{[40]}(x,\xi) = \frac{1}{x-\xi+i0} \pm \frac{1}{x+\xi-i0}\,. \tag{6.60}$$

Further, the "GPDs" $F_{\rho[40]}$ and $\widetilde{F}_{\rho[40]}$ read in the Wandzura-Wilczek approximation

$$\begin{aligned} F_\rho(x,\xi) &\approx \frac{\Delta_\rho}{2\xi}\frac{s}{m}E(x,\xi) - \frac{2\Delta_\rho}{Q^2}(qv)M(x,\xi) \\ &\quad + \int_{-1}^{1}\!du\, G_\rho(u,\xi)W_+(x,u,\xi) + i\epsilon^\perp_{\rho\sigma[40]}\int_{-1}^{1}\!du\,\widetilde{G}^\sigma(u,\xi)W_-(x,u,\xi)\,, \\ \widetilde{F}_\rho(x,\xi) &\approx \frac{\Delta_\rho}{2}\frac{\tilde{s}}{m}\widetilde{E}(x,\xi) - \frac{2\Delta_\rho}{Q^2}(q\tilde{v})\widetilde{H}(x,\xi) \\ &\quad + \int_{-1}^{1}\!du\,\widetilde{G}_\rho(u,\xi)W_+(x,u,\xi) + i\epsilon^\perp_{\rho\sigma[40]}\int_{-1}^{1}\!du\,G^\sigma(u,\xi)W_-(x,u,\xi)\,, \end{aligned} \tag{6.61}$$

where kernels $W_\pm$ are defined as

$$W_\pm(x,u,\xi) = \frac{1}{2}\left[\frac{\theta(x-\xi)\theta(u-x)}{u-\xi} - \frac{\theta(\xi-x)\theta(x-u)}{u-\xi} \pm (\xi \to -\xi)\right], \tag{6.62}$$

and finally

$$\begin{aligned} G^\mu(u,\xi) &\approx v_\perp^\mu M(u,\xi) + \frac{\Delta_\perp^\mu}{2\xi}\frac{s}{m}(u\partial_u + \xi\partial_\xi)E(u,\xi) - \frac{2\Delta_\perp^\mu}{Q^2}(qv)(u\partial_u+\xi\partial_\xi)M(u,\xi), \\ \widetilde{G}^\mu(u,\xi) &\approx \tilde{v}_\perp^\mu \widetilde{H}(u,\xi) + \frac{\Delta_\perp^\mu}{2}\frac{\tilde{s}}{m}(\partial_u u + \xi\partial_\xi)\widetilde{E}(u,\xi) - \frac{2\Delta_\perp^\mu}{Q^2}(q\tilde{v})(u\partial_u+\xi\partial_\xi)\widetilde{H}(u,\xi), \end{aligned} \tag{6.63}$$

where in this equation the subscript "$\perp$" denotes a contraction with $g_{\perp[40]}$, see Eq. (6.59).

The next step will require a lot of Lorentz contractions. Rather immediate is $(\varepsilon^\pm\Delta_\perp) \approx 2\xi(\varepsilon^\pm P)$ with which one finds

$$\begin{aligned} (\varepsilon_L|T_2|\varepsilon^\pm)_{[40]} = -\frac{2\xi}{Q}(\varepsilon^\pm P)\int_{-1}^1 dx \Bigg\{ & C_{[40]}^+(x,\xi)\left[\frac{4\xi}{Q^2}(q'v)M(x,\xi) - \frac{s}{m}E(x,\xi)\right] \\ & \mp C_{[40]}^-(x,\xi)\left[\frac{4\xi}{Q^2}(q'\tilde{v})\widetilde{H}(x,\xi) - \frac{\tilde{s}}{m}\widetilde{E}(x,\xi)\right]\Bigg\} \\ & + G\text{-terms}, \end{aligned} \tag{6.64}$$

where "G-terms" stands for the contributions from the second line of the definition of F_ρ and $\widetilde{F}_\rho$ in Eq. (6.61) (i.e. the terms that involve a convolution with $W_\pm$). To $1/Q^2$ accuracy this coincides with $\frac{4\xi(\varepsilon^\pm P)}{Q}\mathcal{A}_{\pm\pm}$.

It remains to show that "G-terms" reproduce $\mathcal{A}_{0\pm}$. They can be written as follows

$$\begin{aligned} G\text{-terms} = \frac{\xi}{Q}\int_{-1}^1 du \Bigg[& \left(\varepsilon_\mu^\pm G^\mu(u,\xi) + i\varepsilon_\mu^\pm \epsilon_{[40]}^{\perp\mu\nu}\widetilde{G}_\nu(u,\xi)\right)\int_\xi^u dx\, \frac{C_{[40]}^+(u,\xi) - C_{[40]}^-(u,\xi)}{u-\xi} \\ & + \left(\varepsilon_\mu^\pm G^\mu(u,\xi) + i\varepsilon_\mu^\pm \epsilon_{[40]}^{\perp\mu\nu}\widetilde{G}_\nu(u,\xi)\right)\int_{-\xi}^u dx\, \frac{C_{[40]}^+(u,\xi) + C_{[40]}^-(u,\xi)}{u+\xi}\Bigg]. \end{aligned} \tag{6.65}$$

The two integrations over x and u are essentially equivalent to our convolution with C_1, namely

$$G\text{-terms} = \frac{2\xi}{Q}\left[\varepsilon_\mu^\pm C_1 \otimes G^{\mu\,+} + i\varepsilon_\mu^\pm \epsilon_{[40]}^{\perp\mu\nu} C_1 \otimes \widetilde{G}_\nu^+\right], \tag{6.66}$$

where the superscript "+" on $G, \widetilde{G}$ stands for the projection on signature GPDs, see Eq. (6.2). The evaluation of the remaining Lorentz contractions can be done with the help of $(\varepsilon^\pm v_\perp) = (\varepsilon^\pm v) - \frac{4\xi}{Q}(\varepsilon^\pm P)(q'v)$ and similarly with $v \to \tilde{v}$ as well as the following observation (up to $1/Q^2$ corrections)

$$q_\mu = iQ\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon^{+,\,\nu}\varepsilon^{-,\,\rho}\varepsilon_{L[40]}^\sigma\,. \tag{6.67}$$

The last equation is used to show $i\varepsilon_\mu^\pm \epsilon_{[40]}^{\perp\mu\nu}\Delta_{\perp\nu} = \pm 2\xi(\varepsilon^\pm P)$ and $i\varepsilon_\mu^\pm \epsilon_{[40]}^{\perp\mu\nu} v_{\perp\nu} \approx \pm(\varepsilon^\pm v_\perp)$ (similar with $v \to \tilde{v}$). Collecting everything it is straightforward to show that the G-terms are indeed equal to $\mathcal{A}_{0\pm}$.

The situation is different for the twist-4 helicity flip amplitudes, where we find a disagreement between our results and the those given in [40]. Our amplitudes are related to the ones of [40] by

$$(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm}) = -\mathcal{A}_{\mp\pm}\,. \tag{6.68}$$

Instead of quoting the full answer for $A_{[40]}$ we will tentatively focus only on the contribution of say $\widetilde{E}$ and call it $(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})[\widetilde{E}]$. It is given as the sum of two terms

$$(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})[\widetilde{E}] = (\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})_U[\widetilde{E}] + (\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})_W[\widetilde{E}]\,, \tag{6.69}$$

which are in turn given by

$$(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})_U[\widetilde{E}] = \frac{4\xi^2(\varepsilon^{\pm}P)}{Q^2}\int_{-1}^{1} dt\,du\,C^{-}_{[40]}(t,\xi)\,\varepsilon^{\pm}_{\mu} i\epsilon^{\perp\mu\nu}_{[40]}\,\widetilde{G}_{\nu}(u,\xi)U_{+}(u,t,\xi)\,, \tag{6.70}$$

where we retained only the term proportional to $\widetilde{E}$ form $\widetilde{G}_{\nu}$ and

$$\begin{aligned}(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})_W[\widetilde{E}] = \frac{\mp 2\xi^2(\varepsilon^{\pm}P)^2}{Q^2}\frac{\tilde{s}}{m}\int_{-1}^{1} dt\,du\,\big(\ln(t-\xi+i0) - \ln(t+\xi-i0)\big)\times \\ \times W_{-}(u,t,\xi)\big(\xi^2\partial_{\xi}^2 + \xi\partial_{\xi}(2+u\partial_u)\big)\widetilde{E}(u,\xi)\,.\end{aligned} \tag{6.71}$$

U_+ is defined as

$$U_{+}(u,t,\xi) = \frac{1}{2}\left[(t-\xi)\frac{\theta(t-\xi)\theta(u-t)}{(u-\xi)^2} - (t-\xi)\frac{\theta(\xi-t)\theta(t-u)}{(u-\xi)^2} + (\xi\to-\xi)\right]. \tag{6.72}$$

After a couple of lines of calculation one can cast the contributions into

$$\begin{aligned}&(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})_U[\widetilde{E}] = \\ &\quad = \frac{\pm 8\xi^3(P\varepsilon^{\pm})^2}{Q^2}\frac{\tilde{s}}{m}\Big[(\xi\partial_{\xi}^2\xi C_1)\otimes\widetilde{E}^{+} + (\xi\partial_{\xi}C_1)\otimes\big(\xi\partial_{\xi}\widetilde{E}^{+}\big)\Big]\,, \\ &(\varepsilon^{\pm}|A_{[40]}|\varepsilon^{\pm})_W[\widetilde{E}] = \\ &\quad = \frac{\pm 4\xi^3(P\varepsilon^{\pm})^2}{Q^2}\frac{\tilde{s}}{m}\Big[C_1\otimes\xi^2\partial_{\xi}^2\widetilde{E}^{+} + 2C_1\otimes\xi\partial_{\xi}\widetilde{E}^{+} + (\xi\partial_{\xi}C_1)\otimes\big(\xi\partial_{\xi}\widetilde{E}^{+}\big)\Big]\,.\end{aligned} \tag{6.73}$$

Unfortunately the sum "$U+W$" cannot be simplified into a form compatible with Eq. (6.6). A similar observation applies to the contributions of the other GPDs.

6.5. Compton form factors

Nowadays it is a standard procedure to express the amplitudes in terms of *Compton form factors* (CFFs). Loosely defined, a CFF is the nontrivial part of the amplitude tensor, where the Lorentz- and Dirac-structure have been factored out. The functions $\mathbb{V}_{1,2}, \widetilde{\mathbb{V}}_{1,2}$ in Eqs. (6.9) and (6.10) are examples of CFFs. From a pragmatic point of view, the application of the recent results of Belitsky, Müller and Ji (BMJ) [45] requires a formulation in terms of CFFs. To the best of the author's knowledge, BMJ provide the only complete framework, in which our corrections can be incorporated.

In [45] it was suggested to isolate the following Dirac bilinears (the kinematics were converted to our notation)

$$
\begin{aligned}
h &= \frac{(vq')}{2(Pq')}\,, & e &= \frac{(vq')}{2(Pq')} - \frac{s}{2m}\,, \\
\tilde{h} &= \frac{(\tilde{v}q')}{2(Pq')} + \frac{(\tilde{v}\Delta)}{4(Pq')}\,, & \tilde{e} &= -\frac{Q^2\tilde{s}}{8m(Pq')}\,.
\end{aligned} \tag{6.74}
$$

Note that h and $\tilde{h}$ are not to be confused with the double distributions. The above relation can be inverted in favor of the structures appearing in Sec. 6.1 by using $2\xi(Pq') = -(qq')$, $2m\tilde{s} = (\tilde{v}\Delta)$ and $ms = (vP)$

$$
\begin{aligned}
\frac{(vP)}{2m^2} &= h - e\,, & \frac{(vq')}{(qq')} &= -\frac{1}{\xi}h\,, \\
\frac{(\tilde{v}\Delta)}{4m^2} &= -\frac{1}{\xi}\left(1+\frac{t}{Q^2}\right)\tilde{e}\,, & \frac{(\tilde{v}q')}{(qq')} &= -\frac{1}{\xi}\tilde{h} - \frac{1}{\xi}\frac{4m^2}{Q^2}\tilde{e}\,.
\end{aligned} \tag{6.75}
$$

Then one immediately finds

$$
\mathcal{A}_{\pm\pm} = h\,\mathbb{H}_{\pm\pm} + e\,\mathbb{E}_{\pm\pm} \mp \tilde{h}\,\widetilde{\mathbb{H}}_{\pm\pm} \mp \tilde{e}\,\widetilde{\mathbb{E}}_{\pm\pm}\,, \tag{6.76}
$$

where

$$
\begin{aligned}
\mathbb{H}_{\pm\pm} &= \mathbb{V}_1 - \frac{1}{\xi}\mathbb{V}_2\,, & \mathbb{E}_{\pm\pm} &= -\mathbb{V}_1\,, \\
\widetilde{\mathbb{H}}_{\pm\pm} &= \frac{1}{\xi}\widetilde{\mathbb{V}}_2\,, & \widetilde{\mathbb{E}}_{\pm\pm} &= \frac{1}{\xi}\left[\left(1+\frac{t}{Q^2}\right)\widetilde{\mathbb{V}}_1 + \frac{4m^2}{Q^2}\widetilde{\mathbb{V}}_2\right].
\end{aligned} \tag{6.77}
$$

Note that $\mathbb{H}$, $\mathbb{E}$, $\widetilde{\mathbb{H}}$, $\widetilde{\mathbb{E}}$ coincide with $\mathfrak{H}$, $\mathfrak{E}$, $\widetilde{\mathfrak{H}}$, $\widetilde{\mathfrak{E}}$ of Ref. [46] respectively (for the polarization indices "$\pm\pm$" at hand, and the forthcoming ones as well). At this point we would like to give explicit expressions for $\mathbb{H}_{\pm\pm}$ etc. This is of course somewhat redundant, but it can be viewed as *the* main theoretical new result in this work and represents a small milestone in Ref. [46]. First we introduce a new convolution notation,

$$
T \circledast F \equiv \int_{-1}^{1}\frac{d\xi}{2\xi}\,T\!\left(\frac{\xi + x + i0}{2(\xi - i0)}\right)F^{+}(x,\xi)\,. \tag{6.78}
$$

The following definitions

$$
\begin{aligned}
T_0(u) &= \frac{1}{1-u}\,, \\
T_1^{+}(u) &= \frac{(1-2u)\ln(1-u)}{u}\,, \\
T_1^{-}(u) &= -\frac{\ln(1-u)}{u}\,, \\
T_2(u) &= \frac{\mathrm{Li}_2(1) - \mathrm{Li}_2(u)}{1-u} + \frac{\ln(1-u)}{2u}
\end{aligned} \tag{6.79}
$$

allow us to rewrite

$$
\begin{aligned}
C_0 \otimes F^+ &= -T_0 \circledast F\,, & C_0 \otimes \widetilde{F}^+ &= T_0 \circledast \widetilde{F}\,,\\
C_1 \otimes F^+ &= -T_1^- \circledast F\,, & C_1 \otimes \widetilde{F}^+ &= T_1^- \circledast \widetilde{F}\,,\\
(xC_1) \otimes F^+ &= -T_1^+ \circledast F\,, & (xC_1) \otimes \widetilde{F}^+ &= T_1^+ \circledast \widetilde{F}\,,\\
C_2 \otimes F^+ &= T_2^- \circledast F\,, & C_2 \otimes \widetilde{F}^+ &= -T_2^- \circledast \widetilde{F}\,,
\end{aligned} \tag{6.80}
$$

for $F = H, E$ and $\widetilde{F} = \widetilde{H}, \widetilde{E}$. We remind that the operator $T\circledast$ already includes the charge parity projection and flavor summation (weighted with the corresponding fractional electric charge squared). We find

$$
\begin{aligned}
\mathbb{H}_{\pm\pm} &= \left[\left(1 - \frac{t}{2Q^2}\right)T_0 + \frac{t}{Q^2}T_1^- + \frac{2}{\xi Q^2}\left(t + 2|\xi P_\perp|^2 \xi\partial_\xi\right)\xi^2\partial_\xi\, T_2\right]\circledast H + \frac{2t}{Q^2}\xi^2\partial_\xi\, \xi\, T_2 \circledast M\,,\\
\mathbb{E}_{\pm\pm} &= \left[\left(1 - \frac{t}{2Q^2}\right)T_0 + \frac{t}{Q^2}T_1^- + \frac{2}{\xi Q^2}\left(t + 2|\xi P_\perp|^2 \xi\partial_\xi\right)\xi^2\partial_\xi\, T_2\right]\circledast E - \frac{8m^2}{Q^2}\xi^2\partial_\xi \xi\, T_2 \circledast M\,,\\
\widetilde{\mathbb{H}}_{\pm\pm} &= \left[\left(1 - \frac{t}{2Q^2}\right)T_0 + \frac{t}{Q^2}T_1 + \frac{2}{\xi^2 Q^2}\left(t + 2|\xi P_\perp|^2 \xi\partial_\xi\right)\xi^2\partial_\xi\, \xi\, T_2\right]\circledast \widetilde{H} + \frac{2t}{Q^2}\xi\partial_\xi\, T_2 \circledast \widetilde{H}\,,\\
\widetilde{\mathbb{E}}_{\pm\pm} &= \left[\left(1 - \frac{t}{2Q^2}\right)T_0 + \frac{t}{Q^2}T_1 + \frac{2}{\xi^2 Q^2}\left(t + 2|\xi P_\perp|^2 \xi\partial_\xi\right)\xi^2\partial_\xi \xi\, T_2\right]\circledast\\
&\quad \left[\left(1 + \frac{t}{Q^2}\right)\widetilde{E} + \frac{4m^2}{Q^2}\widetilde{H}\right] - \frac{8m^2}{Q^2}\xi\partial_\xi\, T_2 \circledast \widetilde{H}\,.
\end{aligned} \tag{6.81}
$$

Note that Eq. (6.81) contains contributions that are of order $\mathcal{O}(Q^{-4})$. They stem from the rewriting of spinor bilinears, see Eqs. (6.75) and (6.77), and are, technically speaking, beyond our accuracy. However we keep these corrections for the following reason: In phenomenology it has become customary to use cross section formulas in an "exact" fashion, i.e. *not* expanded in Q^2. The higher corrections in (6.81) arise only from a specific choice of the CFF basis. They can therefore be absorbed into the definition of what an "exact" cross section is.

With a little algebra a similar decomposition like in Eq. (6.76) can be achieved for the helicity flip amplitudes. It is convenient to work with the transverse polarizations in the following form, cf. Eqs. (4.57), (4.23), (4.25):

$$
\varepsilon^\pm_\mu = \frac{(g^\perp_{\mu\nu} \pm i\varepsilon^\perp_{\mu\nu})P^\nu}{\sqrt{2}|P_\perp|}\,. \tag{6.82}
$$

From this the following conversion formulas can be derived:

$$
\begin{aligned}
\frac{(\varepsilon^\pm v)}{\sqrt{2}} &= -|P_\perp| h - \frac{m^2}{|P_\perp|}\Big(e - \frac{t}{4m^2}h\Big) \mp \frac{m^2}{\xi|P_\perp|}\Big(\tilde{e} - \frac{t}{4m^2}\tilde{h}\Big)\,,\\
\frac{(\varepsilon^\pm \tilde{v})}{\sqrt{2}} &= -\frac{m^2}{\xi^2|P_\perp|}\Big(\tilde{e} - \frac{t}{4m^2}\tilde{h}\Big) \mp \frac{m^2}{\xi|P_\perp|}\Big(e - \frac{t}{4m^2}h\Big)
\end{aligned} \tag{6.83}
$$

where in the "tilde" sector the γ-matrix identity (3.16) was necessary to establish the second equation. Therefore the parametrizations

$$\begin{aligned}\mathcal{A}_{0\pm} &= h\,\mathbb{H}_{0\pm} + e\,\mathbb{E}_{0\pm} \mp \tilde{h}\,\widetilde{\mathbb{H}}_{0\pm} \mp \tilde{e}\,\widetilde{\mathbb{E}}_{0\pm}\,,\\ \mathcal{A}_{\mp\pm} &= h\,\mathbb{H}_{\mp\pm} + e\,\mathbb{E}_{\mp\pm} \mp \tilde{h}\,\widetilde{\mathbb{H}}_{\mp\pm} \mp \tilde{e}\,\widetilde{\mathbb{E}}_{\mp\pm}\,,\end{aligned} \tag{6.84}$$

are possible and one finds

$$\begin{aligned}\mathbb{H}_{0\pm} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left[\xi\partial_\xi\, T_1^-\circledast H + \frac{t}{Q^2}\partial_\xi\,\xi\, T_1^-\circledast M\right] + \frac{t}{\sqrt{2}Q|P_\perp|}T_1^-\circledast\big(\widetilde{H} - \xi M\big)\\ \mathbb{E}_{0\pm} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\xi\partial_\xi\, T_1^-\circledast E + \frac{4m^2}{\sqrt{2}Q|P_\perp|}T_1^-\circledast\big(\xi M - \widetilde{H}\big)\\ \widetilde{\mathbb{H}}_{0\pm} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left(1 + \frac{t}{Q^2}\right)\partial_\xi\,\xi\, T_1^-\circledast\widetilde{H} + \frac{t}{\sqrt{2}Q|\xi P_\perp|}T_1^-\circledast\big(\xi M - \widetilde{H}\big)\\ \widetilde{\mathbb{E}}_{0\pm} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\left(1 + \frac{t}{Q^2}\right)\partial_\xi\,\xi\, T_1^-\circledast\left(\widetilde{E} + \frac{4m^2}{Q^2}\widetilde{H}\right) + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|}T_1^-\circledast\big(\widetilde{H} - \xi M\big),\end{aligned} \tag{6.85}$$

where we remind that $M = H + E$ and

$$\begin{aligned}\mathbb{H}_{\mp\pm} &= \frac{4|\xi P_\perp|^2}{Q^2}\left(\xi\partial_\xi^2\,\xi\, T_1^+\circledast H + \frac{t}{Q^2}\partial_\xi^2\,\xi^2\, T_1^+\circledast M\right) + \frac{2t}{Q^2}\big(\xi^2\partial_\xi\,\xi\, T_1^+\circledast M + \xi\partial_\xi\,\xi\, T_1^-\circledast\widetilde{H}\big)\,,\\ \mathbb{E}_{\mp\pm} &= \frac{4|\xi P_\perp|^2}{Q^2}\xi\partial_\xi^2\,\xi\, T_1^+\circledast E - \frac{8m^2}{Q^2}\big(\xi^2\partial_\xi\,\xi\, T_1^+\circledast M + \xi\partial_\xi T_1\circledast\widetilde{H}\big)\,,\\ \widetilde{\mathbb{H}}_{\mp\pm} &= -\frac{4|\xi P_\perp|^2}{Q^2}\left(1 + \frac{t}{Q^2}\right)\partial_\xi^2\,\xi^2\, T_1^-\circledast\widetilde{H} - \frac{2t}{Q^2}\big(\partial_\xi\,\xi\, T_1^-\circledast\widetilde{H} + \xi\partial_\xi\,\xi\, T_1^+\circledast M\big)\,,\\ \widetilde{\mathbb{E}}_{\mp\pm} &= \frac{4|\xi P_\perp|^2}{Q^2}\left(1 + \frac{t}{Q^2}\right)\partial_\xi^2\,\xi^2\, T_1^-\circledast\left(\widetilde{E} - \frac{4m^2}{Q^2}\widetilde{H}\right) + \frac{8m^2}{Q^2}\big(\xi\partial_\xi\,\xi\, T_1^+\circledast M + \partial_\xi\,\xi\, T_1^-\circledast\widetilde{H}\big)\,.\end{aligned} \tag{6.86}$$

As one immediately reads off, the number of *independent* CFFs is twelve, since $\mathbb{F}_{ab} = \mathbb{F}_{-a,-b}$ for $a, b \in \{0, \pm\}$ and $\mathbb{F} \in \{\mathbb{H}, \mathbb{E}, \widetilde{\mathbb{H}}, \widetilde{\mathbb{E}}\}$. Each of them is complex-valued and the basic dispersion relation for a GPD F without D-term or pion-pole reads

$$T\circledast F = \frac{1}{\pi}\int_0^1 dx\,\frac{x + \xi + \sigma(x - \xi)}{\xi^2 - x^2 - i0}(\mathrm{Im}\,T\circledast F)(x)\,, \tag{6.87}$$

where $F \in \{H, E, \widetilde{H}, \widetilde{E}\}$ with its accompanying signature factor σ and $T \in \{T_0, T_1^\pm, T_2\}$. The proof of this equation is straightforward for the imaginary part and the real part follows from (6.29) and (6.30). Note that there is a small subtlety for T_1^+, which does not vanish at infinity. Therefore the arcs at infinity, see Fig. 6.1, give a nonvanishing contribution. As can be shown this boundary term is of the form const./ξ, which never manifests itself in the CFFs, since $T_1^+\circledast\ldots$ is always preceded by either $\partial_\xi\xi$ or $\partial_\xi^2\xi^2$. These differential operators put the term to zero, and one can effectively treat Eq. (6.87) as if that subtraction was nonexistent. Furthermore, it is also possible to formulate dispersion relations on the level of CFFs themselves. In a complete treatment one has to take into account subtraction terms related to D-term and pion-pole contributions. Additional subtractions can occur from ξ-derivatives, cf. [46] for more details.

An additional observation concerns the target mass corrections [46]. Looking at the results in the DD representation, see Eqs. (5.36), (5.47), (5.115), (5.116), (5.128) and (5.129), one realizes that the corrections $m/Q, m^2/Q^2$ are all absorbed in $P_\perp$, by

$$|P_\perp|^2 = \frac{1-\xi^2}{4\xi^2}(t_{\rm min} - t)\,, \quad t_{\rm min} = -\frac{4\xi^2 m^2}{1-\xi^2}\,. \tag{6.88}$$

Since $t \leq t_{\rm min}$ the finite-t effects always overcompensate the finite-m effects. Additional mass corrections can arise from rewriting the Dirac bilinears, as present in Eqs. (6.81), (6.85) and (6.86). In the case of a scalar target such terms are absent.

Let us close this chapter by outlining another representation of our results: in recent years the evaluation of the convolutions based on Mellin-Barnes integrals became quite popular, see [47–49]. As an essential ingredient to implement the power corrections into such a framework, one needs the so-called *conformal moments* of the kernels (6.79). One of the strengths of this approach is that QCD evolution can be incorporated rather easily. At leading order (and in certain schemes also at next-to-leading order) the conformal moments evolve autonomously [50]. The j-th conformal moment $T[j]$ of a kernel T is defined as

$$T[j] = \int_0^1 du\, p_j(u) T(u) \tag{6.89}$$

with

$$p_j(u) = 2u(1-u) C_j^{\frac{3}{2}}(2u-1)\,, \tag{6.90}$$

where $C_j^{\frac{3}{2}}(u)$ is the j-th Gegenbauer polynomial with index 3/2. The results for the kernels (6.79) are

$$\begin{aligned} T_0[j] &= 1\,, \\ T_1^+[j] &= \frac{2+(1+j)(2+j)}{j(1+j)(2+j)(3+j)}\,, \\ T_1^-[j] &= \frac{1}{(1+j)(2+j)}\,, \\ T_2[j] &= \frac{2+(1+j)(2+j)}{(1+j)^2(2+j)^2}\,. \end{aligned} \tag{6.91}$$

Let us sketch the derivation of these formulas on the contribution, which arises from the dilogarithmic terms. Therefore consider the example kernel

$$T_{\rm Li}(u) = \frac{{\rm Li}_2(1) - {\rm Li}_2(u)}{1-u}\,, \tag{6.92}$$

and by definition

$$T_{\rm Li}[j] = 2\int_0^1 du\, u\, C_j^{\frac{3}{2}}(2u-1)\big({\rm Li}_2(1) - {\rm Li}_2(u)\big)\,. \tag{6.93}$$

Instead of (6.93) we calculate a more general integral

$$f(t) = 2\int_0^1 du\, u \frac{{\rm Li}_2(1) - {\rm Li}_2(u)}{(1-2(2u-1)t + t^2)^{3/2}}\,, \tag{6.94}$$

which contains the answer for $T_{\rm Li}[j]$ through the relation of the Gegenbauer polynomials to their generating function

$$\frac{1}{(1-2st+t^2)^{3/2}} = \sum_{j=0}^{\infty} C_j^{\frac{3}{2}}(s)t^j \,. \tag{6.95}$$

Employing the definition of the Spence function (5.94) one finds after a short calculation

$$f(t) = -\frac{4}{b^2}(1+2a\partial_a)\int_0^1 dv\,\frac{\ln(1-v)}{v}\sqrt{a+bv} - \frac{8\sqrt{a}\,\mathrm{Li}_2(1)}{b^2}\,, \tag{6.96}$$

where $a = (1+t)^2$ and $b = -4t$. The remaining integral can be calculated with conventional methods, the intermediate steps however are too lengthy to be presented here, so we only give the answer:

$$f(t) = \frac{(1+t)\,\mathrm{Li}_2(t) - (1-t)\ln(1-t) - 2t}{t^2}\,. \tag{6.97}$$

This expression is an analytical function of t inside the complex unit-disk, which can explicitly be seen by power-expanding the result in t using

$$\mathrm{Li}_2(t) = \sum_{j=1}^{\infty}\frac{t^j}{j^2}\,, \quad \ln(1-t) = -\sum_{j=1}^{\infty}\frac{t^j}{j}\,. \tag{6.98}$$

Then one gets

$$f(t) = \sum_{j=0}^{\infty} t^j\left(\frac{1}{(j+2)^2} + \frac{1}{(j+1)^2} + \frac{1}{j+2} - \frac{1}{j+1}\right), \tag{6.99}$$

from which we read off

$$T_{\mathrm{Li}}[j] = \frac{1+(1+j)(2+j)}{(1+j)^2(2+j)^2}\,. \tag{6.100}$$

The full set of conformal moments (6.91) is calculable along the same lines.

7. Phenomenology: confronting physical observables

In a recent publication [45] Belitsky, Müller and Ji presented cross section formulas for DVCS for all polarization configurations of the scattering particles, most importantly for us, including the photon helicity flip amplitudes. The observables were calculated in lowest order of quantum electrodynamics (QED) and are parametrized in terms of the Dirac and Pauli form factors and a particular set of Compton form factors. Ref. [45] is the correct place to insert our results because the process kinematics are treated exactly, i.e. unexpanded in $1/Q$. It requires a little preparatory work to convert the notations, which is done in the next section.

7.1. A different CFF basis: BMJ formulation

Before we are ready to apply the framework of BMJ, we need to aim at a conversion of the CFF basis from Sec. 6.5 to the one from [45]. We will point out where the conventions differ, and use the label "BMJ" to indicate the "external" notations. If a quantity is unambiguous, we will omit that flag.

BMJ define the hadronic Compton tensor as

$$T^{\mathrm{BMJ}}_{\mu\nu} = i \int d^4x \, e^{i(q+q')x/2} \, \langle p', s' | \, \mathrm{T}\{ j_\mu(x/2) j_\nu(-x/2) \} \, | p, s \rangle \, , \tag{7.1}$$

from which we easily read off the relation, cf. (4.33)

$$T^{\mathrm{BMJ}}_{\mu\nu} = \mathcal{A}_{\nu\mu} \, . \tag{7.2}$$

They further define helicity amplitudes $\mathcal{T}^{\mathrm{BMJ}}_{a\pm}$ by the contraction

$$\mathcal{T}^{\mathrm{BMJ}}_{a\pm} \equiv (-1)^{a-1} \big(\varepsilon^{\mu}_{2,\mathrm{BMJ}}(\pm) \big)^* T^{\mathrm{BMJ}}_{\mu\nu} \varepsilon^{\nu}_{1,\mathrm{BMJ}}(a) \, , \quad a \in \{0, \pm\} \tag{7.3}$$

with the polarization vectors

$$\begin{aligned}
\varepsilon^{\mu}_{1,\mathrm{BMJ}}(0) &= -\frac{1}{Q\sqrt{1+\gamma^2}} q^\mu - \frac{2x_B}{Q\sqrt{1+\gamma^2}} p^\mu \, , \\
\varepsilon^{\mu}_{1,\mathrm{BMJ}}(\pm) &= \frac{\sqrt{1+\gamma^2}}{\sqrt{2}\widetilde{K}} \left[\Delta^\mu - \frac{\gamma^2(Q^2-t) - 2x_B t}{2Q^2\,(1+\gamma^2)} q^\mu + x_B \frac{Q^2 - t + 2x_B t}{Q^2(1+\gamma^2)} p^\mu \right] \pm \frac{x_B}{\sqrt{2}\widetilde{K}} \frac{i\varepsilon_{pq\Delta}{}^{\mu}}{Q^2} \, , \\
\varepsilon^{\mu}_{2,\mathrm{BMJ}}(\pm) &= \frac{1 + \frac{\gamma^2}{2} \frac{Q^2+t}{Q^2+x_B t}}{\sqrt{2}\widetilde{K}} \left[\Delta^\mu - \frac{\gamma^2(Q^2-t) - 2x_B t}{2Q^2(1+\gamma^2)} q^\mu + x_B \frac{Q^2 - t + 2x_B t}{Q^2(1+\gamma^2)} p^\mu \right] \\
&\quad + \frac{\widetilde{K}}{\sqrt{2}(1+\gamma^2)(Q^2+x_B t)} \left[\gamma^2 q^\mu - 2x_B p^\mu \right] \pm \frac{x_B}{\sqrt{2}\widetilde{K}} \frac{i\varepsilon_{pq\Delta}{}^{\mu}}{Q^2} \, .
\end{aligned} \tag{7.4}$$

At this point we already took the opportunity to correct for the different Levi-Civita convention (BMJ are using $\varepsilon^{0123} = 1$ which differs in sign from the convention employed here). The Bjorken scaling variable is identical to ours, $x_B = Q^2/\big(2(pq)\big)$, see Eq. (4.14). These formulas include a shorthand for $\varepsilon_{pq\Delta}{}^{\mu} = \varepsilon^{\nu\rho\sigma\mu} p_\nu q_\rho \Delta_\sigma$ and

$$\gamma = \frac{2x_B m}{Q}\,, \tag{7.5}$$

as well as

$$\widetilde{K} = \sqrt{x_B \bar{x}_B + \frac{\gamma^2}{4}}\sqrt{\frac{(t_{\min} - t)(t_{\max} - t)}{Q^4}}\,, \tag{7.6}$$

with

$$t_{\min} = -Q^2 \frac{2\bar{x}_B\big(1 - \sqrt{1+\gamma^2}\big) + \gamma^2}{4x_B\bar{x}_B + \gamma^2}\,, \quad t_{\max} = -Q^2 \frac{2\bar{x}_B\big(1 + \sqrt{1+\gamma^2}\big) + \gamma^2}{4x_B\bar{x}_B + \gamma^2}\,. \tag{7.7}$$

Here $t_{\min}$ coincides with $t_{\min}$ from Eq. (4.17). A more compact representation of $\widetilde{K}$ can be given by eliminating γ though $P_\perp$:

$$\widetilde{K} = x_B\left(1 + \frac{t}{Q^2}\right)|P_\perp|\,. \tag{7.8}$$

In order to make a connection between $\mathcal{T}^{\mathrm{BMJ}}_{a\pm}$ and our $\mathcal{A}_{a\pm}$, we use the definition of the BMJ polarization basis (7.4) and express the process momenta through our polarization vectors. One needs the following identity

$$p_\mu = -\frac{1}{2x_B} q_\mu + \frac{Q}{2x_B}\frac{Q^2 - t + 2x_B t}{Q^2 + t}\varepsilon^0_\mu + \frac{|P_\perp|}{\sqrt{2}}(\varepsilon^+_\mu + \varepsilon^-_\mu)\,. \tag{7.9}$$

In anticipation of contractions with the amplitude tensor, we can drop terms proportional to q' in $\varepsilon^{\mu}_{2,\mathrm{BMJ}}(\pm)$ due to the Ward identities (4.35). The remaining terms proportional to q drop out exactly using Eq. (4.49) at this level, which ensures that the unphysical amplitudes $\mathcal{A}_{a0}$ do not contribute to any observable later on. Next, $i\varepsilon_{pq\Delta\mu}$ can have only transverse components and by direct calculation using (4.50) or (4.57)

$$i\varepsilon_{pq\Delta\mu} = \frac{Q^2\widetilde{K}}{\sqrt{2}x_B}(\varepsilon^-_\mu - \varepsilon^+_\mu) \tag{7.10}$$

and thus

$$\varepsilon^{\mu}_{2,\mathrm{BMJ}}(\pm) = \varepsilon^{\mp\,\mu}\,, \tag{7.11}$$

where "equality" is understood up to irrelevant terms proportional to $q'^{\,\mu}$. Then the final state contraction is easily written down

$$\mathcal{T}^{\mathrm{BMJ}}_{a\pm} = (-1)^a \varepsilon^{\mathrm{BMJ},\nu}_1(a)\big(\varepsilon^{\pm}_\nu \mathcal{A}_{\pm\pm} + \varepsilon^0_\nu \mathcal{A}_{0\pm} + \varepsilon^{\mp}_\nu \mathcal{A}_{\mp\pm}\big)\,. \tag{7.12}$$

The initial state polarizations can be treated in a somewhat similar way, ignoring terms proportional to q for the same reason as above. A short calculation yields

$$
\begin{aligned}
\varepsilon^{\mu}_{1,\mathrm{BMJ}}(0) &= -\kappa\varepsilon^{0\,\mu} - \kappa_0(\varepsilon^{+\,\mu} + \varepsilon^{-\,\mu})\,,\\
\varepsilon^{\mu}_{1,\mathrm{BMJ}}(\pm) &= \varepsilon^{\pm\,\mu} + \frac{\kappa}{2}(\varepsilon^{+\,\mu} + \varepsilon^{-\,\mu}) + \kappa_0\varepsilon^{0\,\mu}\,,
\end{aligned}
\tag{7.13}
$$

where

$$
\begin{aligned}
\kappa_0 &= \frac{\sqrt{2}Q\widetilde{K}}{\sqrt{1+\gamma^2}(Q^2+t)} = \mathcal{O}(1/Q)\,,\\
\kappa &= \frac{Q^2 - t + 2x_B t}{\sqrt{1+\gamma^2}(Q^2+t)} - 1 = \mathcal{O}(1/Q^2)\,.
\end{aligned}
\tag{7.14}
$$

Finally we obtain

$$
\begin{aligned}
\mathcal{T}^{\mathrm{BMJ}}_{\pm\pm} &= \mathcal{A}_{\pm\pm} + \frac{\kappa}{2}\big(\mathcal{A}_{\pm\pm} + \mathcal{A}_{\mp\pm}\big) - \kappa_0\mathcal{A}_{0\pm}\,,\\
\mathcal{T}^{\mathrm{BMJ}}_{0\pm} &= -(\kappa+1)\mathcal{A}_{0\pm} + \kappa_0(\mathcal{A}_{\pm\pm} + \mathcal{A}_{\mp\pm})\,,\\
\mathcal{T}^{\mathrm{BMJ}}_{\mp\pm} &= \mathcal{A}_{\mp\pm} + \frac{\kappa}{2}\big(\mathcal{A}_{\pm\pm} + \mathcal{A}_{\mp\pm}\big) - \kappa_0\mathcal{A}_{0\pm}\,.
\end{aligned}
\tag{7.15}
$$

BMJ continue to define CFFs with respect to the Dirac bilinears in Eq. (6.74),

$$
\mathcal{T}^{\mathrm{BMJ}}_{a\pm} = h\,\mathcal{H}_{a\pm} + e\,\mathcal{E}_{a\pm} \mp \tilde{h}\,\widetilde{\mathcal{H}}_{a\pm} \mp \tilde{e}\,\widetilde{\mathcal{E}}_{a\pm}\,.
\tag{7.16}
$$

Since we have already performed such a decomposition in Sec. 6.5, the final relations between our CFFs and those of BMJ can be obtained from Eq. (7.15) by simply replacing the amplitudes with the CFFs. In more detail this means

$$
\begin{aligned}
\mathcal{F}_{\pm\pm} &= \mathbb{F}_{\pm\pm} + \frac{\kappa}{2}\big(\mathbb{F}_{\pm\pm} + \mathbb{F}_{\mp\pm}\big) - \kappa_0\mathbb{F}_{0\pm}\,,\\
\mathcal{F}_{0\pm} &= -(\kappa+1)\mathbb{F}_{0\pm} + \kappa_0(\mathbb{F}_{\pm\pm} + \mathbb{F}_{\mp\pm})\,,\\
\mathcal{F}_{\mp\pm} &= \mathbb{F}_{\mp\pm} + \frac{\kappa}{2}\big(\mathbb{F}_{\pm\pm} + \mathbb{F}_{\mp\pm}\big) - \kappa_0\mathbb{F}_{0\pm}\,.
\end{aligned}
\tag{7.17}
$$

for $\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$, $\mathbb{F} \in \{\mathbb{H}, \mathbb{E}, \widetilde{\mathbb{H}}, \widetilde{\mathbb{E}}\}$. From these equations and $\mathbb{F}_{ab} = \mathbb{F}_{-a,-b}$ it is explicit, that

$$
\begin{aligned}
\mathcal{F}_{++} &= \mathcal{F}_{--}\,,\\
\mathcal{F}_{0\pm} &= \mathcal{F}_{0\mp}\,,\\
\mathcal{F}_{\mp\pm} &= \mathcal{F}_{\pm\mp}\,,
\end{aligned}
\tag{7.18}
$$

which is again a consequence of parity.

It shall be stressed that Eq. (7.17) contains corrections suppressed by $1/Q^3$ or higher, which we can keep for the same reason as we did in Sec. 6.5. The factors κ, κ_0 are "geometric" (in the sense that they originate only from a change of basis) and can be thought of as being part of the unexpanded cross section.

7.2. Model and conventions

In order to estimate the impact of the power corrections to DVCS, one needs an ansatz for the GPDs. We select a model by Kroll, Moutarde and Sabatie [51], which is an updated version of the model from Goloskokov and Kroll [52,53]. We will refer to it as the *GK12* model. It is based on the double distribution ansatz by Radyushkin [54,55] for which a GPD F^q of quark flavor q is given by

$$F^q(x,\xi,t) = \int d\beta\, d\alpha\, \delta(\beta+\xi\alpha-x)\, f^q(\beta,\alpha,t) + \delta_{F^q\widetilde{E}^q}\frac{1}{|\xi|}\varphi^q_\pi\left(\frac{x}{\xi},t\right)\theta(\xi^2-x^2)\,, \qquad (7.19)$$

where the second term is the pion-pole contribution for $\widetilde{E}^q$, taken as in [56]. A D-term for H^q and E^q is neglected. The DD $f^q(\beta,\alpha,t)$ has the functional form of a product of the forward limit with a weight function

$$f^q(\beta,\alpha,t) = h^q(\beta)e^{b^q t}|\beta|^{-a^q t}\frac{\Gamma(2n^q+2)}{2^{2n^q+1}(\Gamma(n^q+1))^2}\frac{\left[(1-|\beta|)^2-\alpha^2\right]^{n^q}}{(1-|\beta|^2)^{2n^q+1}}\,. \qquad (7.20)$$

The parameter n^q is taken as 1 for valence and 2 for sea quarks. The normalization is chosen is such a way that h^q constitutes the forward limit

$$F^q(x,\xi=0,t=0) = h^q(x)\,. \qquad (7.21)$$

This allows for a fit to the forward unpolarized (for H^q) and polarized (for $\widetilde{H}^q$) parton distribution functions, which are accessible in DIS. Kroll et al. have aimed at the models [57] and [58] respectively through linear combinations of the ansatz

$$h^q(\beta) \sim \beta^{c^q}(1-\beta)^{d^q}\,. \qquad (7.22)$$

Note that (7.22) is only schematic, i.e. up to support restrictions, symmetrizations, etc. This ansatz is also applied to E and $\widetilde{E}$. The values of the individual parameters shall not be repeated here and can be found in Ref. [51]. QCD evolution is partially taken into account through a dependence of these parameters on the factorization scale μ^2, which is always taken to be equal to the photon virtuality Q^2.

Let us now take this model and try to compare leading twist CFFs to the CFFs including power corrections. However, a “leading twist” framework is not uniquely defined, it is rather a convention. For example, adhering to the factors arising from Dirac bilinears in Sec. 6.5 or the geometric factors of 7.1 without re-expanding the CFFs is one convention. Dropping all terms that are beyond the desired order in $1/Q$ or higher would be another. Furthermore the definition of ξ in terms of x_B, see Eq. (4.15),

$$\xi = \frac{x_B}{2-x_B} + \mathcal{O}(t/Q^2) \qquad (7.23)$$

is intrinsically tied to a certain power accuracy. Out of the multitude of conventions, we have taken three representatives [46]:

(i) “twist-4”: Following the philosophy of absorbing certain kinematical functions into the definition of the cross sections, the CFFs $\mathcal{F}_{ab}$ ($a=0,\pm$ and $b=\pm$) are taken as in Eq. (7.17) with κ, κ_0 given in Eq. (7.14) and $\mathbb{F}_{ab}$ in Eqs. (6.81), (6.85), (6.86). Leaving the geometrical and spinorial factors unexpanded has the advantage that all observables obtained from [45] would be identical to a recalculation of [45] in a different basis, namely the one given by the original Eqs. (6.6)–(6.10). For the skewness we take

$$\xi = \xi_{\mathrm{BMP}} \equiv \frac{x_B\left(1+\frac{t}{Q^2}\right)}{2-x_B\left(1-\frac{t}{Q^2}\right)} \,. \tag{7.24}$$

The label "BMP" refers to the publications [42, 44], in which this convention was proposed.

(ii) "$\mathrm{LT}_{\mathrm{BMP}}$": The amplitudes $\mathcal{A}_{ab}$ are truncated in Q^2 *except* for the factors arising from the Dirac bilinears in Eq. (6.75), i.e. on the level of CFFs

$$\begin{aligned}
&\mathbb{F}_{++} = T_0 \circledast F\,, \quad \mathbb{F} \in \{\mathbb{H}, \mathbb{E}, \widetilde{\mathbb{H}}\}\,,\\
&\widetilde{\mathbb{E}}_{++} = \left(1+\frac{t}{Q^2}\right) T_0 \circledast \widetilde{E} + \frac{4m^2}{Q^2} T_0 \circledast \widetilde{H}\,,\\
&\mathbb{F}_{0+} = 0\,,\\
&\mathbb{F}_{-+} = 0\,.
\end{aligned} \tag{7.25}$$

The change of basis is also done without truncation

$$\mathcal{F}_{++} = \left(1+\frac{\kappa}{2}\right)\mathbb{F}_{++}\,, \quad \mathcal{F}_{0+} = \kappa_0 \mathbb{F}_{++}\,, \quad \mathcal{F}_{-+} = \frac{\kappa}{2}\mathbb{F}_{++}\,, \tag{7.26}$$

and ξ equal to the "BMP" convention,

$$\xi = \xi_{\mathrm{BMP}}\,. \tag{7.27}$$

Note that here helicity flip amplitudes appear in the $\mathcal{F}$-basis, while they are absent in the $\mathbb{F}$-basis. This is, roughly speaking, a consequence of Lorentz transformations. BMP and BMJ define helicities, and therefore (non-)conservation of the same, in their respective frame and the relation is given in Sec. 7.1. Formally both descriptions coincide in the Bjorken limit, but on the other hand κ, κ_0 can become numerically important for present experimental kinematics. This property has also been noticed in [59]. $\mathrm{LT}_{\mathrm{BMP}}$ can be regarded as a slight but incomplete advance into the higher twist domain. As we will see below, in some situations $\mathrm{LT}_{\mathrm{BMP}}$ captures much of the numerical impact of "twist-4". From the technical point of view, only convolutions with T_0 have to be calculated and thus the implementation of $\mathrm{LT}_{\mathrm{BMP}}$ in existing fitting codes should be rather straightforward.

(iii) "$\mathrm{LT}_{\mathrm{KM}}$": A widely used leading twist framework is the one dubbed "KM" (Kumerički and Müller [48,49]). In our context the CFFs are defined by "truncating everything from Eq. (7.17)", i.e. $\kappa_0 \to 0$, $\kappa \to 0$ as well as neglecting all terms $\sim 1/Q$ in the expressions for $\mathbb{F}$. To be precise

$$\mathcal{F}_{++} = T_0 \circledast F\,, \quad \mathcal{F}_{0+} = 0\,, \quad \mathcal{F}_{-+} = 0\,, \quad \mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}\,. \tag{7.28}$$

The skewness is taken to be

$$\xi = \xi_{\mathrm{KM}} \equiv \frac{x_B}{2-x_B}\,. \tag{7.29}$$

One can regard the KM convention as the "standard" leading twist framework, it does

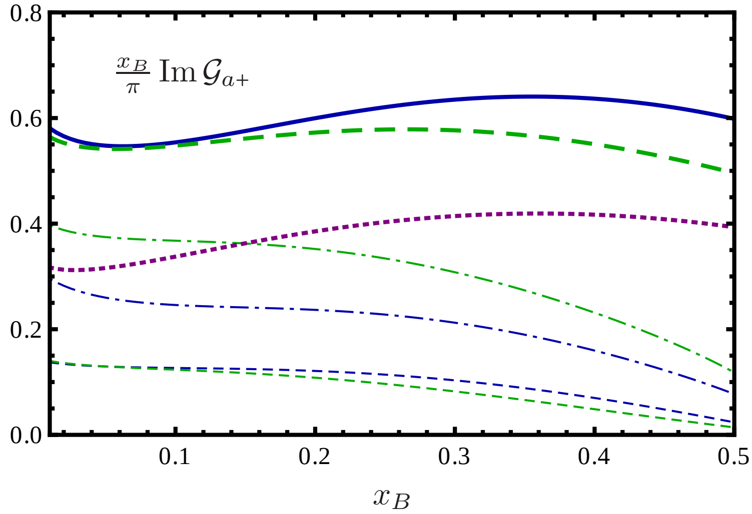

Figure 7.1.: Imaginary part of the electric CFFs $\mathrm{Im}\,\mathcal{G}_{a+}$ (times x_B/π) at $Q^2 = 1.5\,\mathrm{GeV}^2$, $t = -0.375\,\mathrm{GeV}^2$ vs. x_B to different power accuracies. The twist-4 convention is shown by the blue curves for $a = +$ (thick, solid), $a = 0$ (thin, dot-dashed), $a = -$ (thin, dashed). $\mathrm{LT_{BMP}}$ is depicted by the green curves $a = +$ (thick, dashed), $a = 0$ (thin, dot-dashed), $a = -$ (thin, dashed). For $\mathrm{LT_{KM}}$ the only nonzero CFF is given for $a = +$ (purple, dotted).

not differ much from customary conventions, like VGG [60] (Vanderhaeghen-Guichon-Guidal) or the one used by Kroll *et. al.* [51]. $\mathrm{LT_{KM}}$ can be thought of as "state of the art" before the developments in [42,44–46].

To gain some insight into the difference between the three scenarios, let us consider the following "electric" combinations

$$\mathcal{G}_{a+} = \mathcal{H}_{a+} + \frac{t}{4m^2}\mathcal{E}_{a+}\,,$$
$$\widetilde{\mathcal{G}}_{a+} = \widetilde{\mathcal{H}}_{a+} + \frac{t}{4m^2}\widetilde{\mathcal{E}}_{a+}\,. \tag{7.30}$$

For illustration we evaluate $\mathcal{G}_{a+}$ and $\widetilde{\mathcal{G}}_{a+}$ numerically with the *GK12* model for $Q^2 = 1.5\,\mathrm{GeV}^2$ and $t = -0.375\,\mathrm{GeV}^2$ and show the imaginary parts (times x_B/π) as functions of x_B in Figs. 7.1 and 7.2. A detailed discussion on how the individual convolutions combine into the resulting curves can be found in [46] and shall not be repeated here.

From Fig. 7.1 one can see that for the helicity conserving $\mathrm{Im}\,\mathcal{G}_{++}$ the numerical difference between the two leading twist approximations is already quite significant (green-thick-dashed vs. purple-thick-dotted curve). Apart from comparably large prefactors (like κ and κ_0) this is also due to the difference between the definitions of the skewness parameter, namely $\xi_{\mathrm{BMP}} < \xi_{\mathrm{KM}}$. Generally this probes the GPDs in different regions. For instance $\mathrm{Im}\,T_0{\otimes}G = \pi G(\xi,\xi)$ (with $G = H + t/(4m^2)E$) is sensitive to the GPDs on the crossover line. Going from $\xi = \xi_{\mathrm{KM}}$ to $\xi = \xi_{\mathrm{BMP}}$ can produce a large enhancement since $G(\xi,\xi)$ increases rapidly with decreasing ξ. On the other hand, twist-4 and $\mathrm{LT_{BMP}}$ differ only moderately over a wide range in x_B (blue-thick-solid vs. green-thick-dashed curve). The longitudinal-to-transverse flip $\mathrm{Im}\,\mathcal{G}_{0+}$ is much larger in the $\mathrm{LT_{BMP}}$ sector, where it is proportional to $\kappa_0\mathbb{F}_{++}$, while

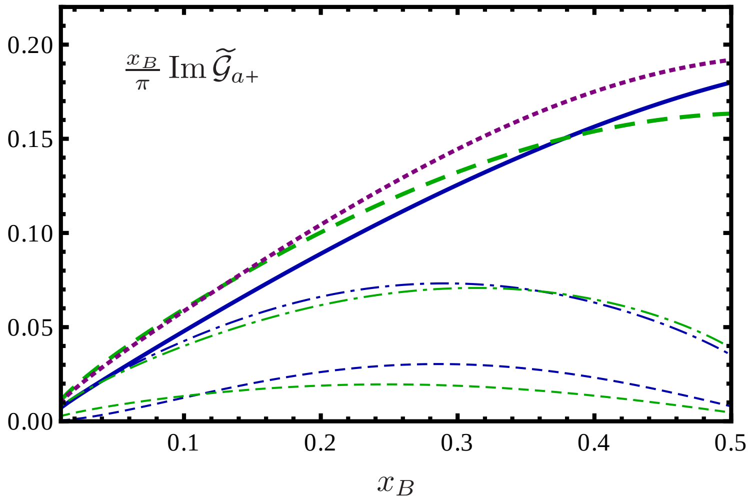

Figure 7.2.: Imaginary part of the electric CFFs $\mathrm{Im}\,\widetilde{\mathcal{G}}_{a+}$ (times x_B/π) vs. x_B to different power accuracies. The kinematics and curve styles are identical to Fig. 7.1.

the twist-4 expression is reduced by the additional contributions in Eq. (7.17) (thin-dash-dotted green/blue). Both curves are quite sizeable and comparable in magnitude to the $\mathrm{LT}_{\mathrm{KM}}$ helicity conserving CFF. The transverse-to-transverse flip CFFs (thin-short-dashed blue/green) appear to be almost the same in the two frameworks, which can be traced back to a cancellation between the contributions of $\mathbb{F}_{-+}$ and $\mathbb{F}_{0+}$ in Eq. (7.17). Overall they can be considered as rather small. Finally note that $\mathrm{Im}\,\mathcal{G}_{0+}$ and $\mathrm{Im}\,\mathcal{G}_{-+}$ vanish at $x_B \sim 0.55$, where one "hits" the kinematical boundary $t = t_{\mathrm{min}}$. The BMJ basis was specifically designed to exhibit the behavior

$$\begin{aligned}\mathcal{G}_{0+} &\sim (t_{\mathrm{min}} - t)^{\frac{1}{2}}\,,\\ \mathcal{G}_{-+} &\sim (t_{\mathrm{min}} - t)^{1}\,.\end{aligned} \tag{7.31}$$

This property guarantees that certain harmonics in the cross section vanish in the limit $t \to t_{\mathrm{min}}$.

In the axial sector all three conventions result in CFFs $\mathrm{Im}\,\widetilde{\mathcal{G}}_{a+}$ that are roughly of the same size. This is basically due to a cancellation between $\widetilde{\mathcal{H}}$ and $\widetilde{\mathcal{E}}$. Note that individual CFFs, especially $\widetilde{\mathcal{E}}_{++}$, can differ vastly due to the large prefactor $4m^2/Q^2$ stemming from Eq. (6.77). Again we found that the longitudinal-to-transverse CFFs $\mathrm{Im}\,\widetilde{\mathcal{G}}_{0+}$ can become relatively large, while the two-unit helicity flip CFFs $\mathrm{Im}\,\widetilde{\mathcal{G}}_{-+}$ remain rather small. Just like in the previous case $\widetilde{\mathcal{G}}_{0+}$ and $\widetilde{\mathcal{G}}_{-+}$ vanish as $(t_{\mathrm{min}} - t)^{\frac{1}{2}}$ and $(t_{\mathrm{min}} - t)^{1}$ at the phase space boundary. In addition $x_B\,\mathrm{Im}\,\widetilde{\mathcal{G}}_{a+}$ also vanish as $x_B \to 0$, since $\widetilde{\mathcal{G}}_{a+}$ depends only on $\widetilde{H}$ and $\widetilde{E}$, which have only a mild pomeron behavior at small ξ: *GK12* assumes a "valence-like" parametrization which gives $\widetilde{H}, \widetilde{E} \sim \xi^{-0.31}$ for the specified kinematics.

We would like to stress here, that the actual impact of the twist-4 sector should be discussed in view of more direct observables like cross sections, asymmetries etc. Depending on the observable, it is in general not safe to say that large differences in CFFs automatically translate into large differences in the observable under consideration. Therefore we continue to investigate the effect on actual measurable quantities in the next section.

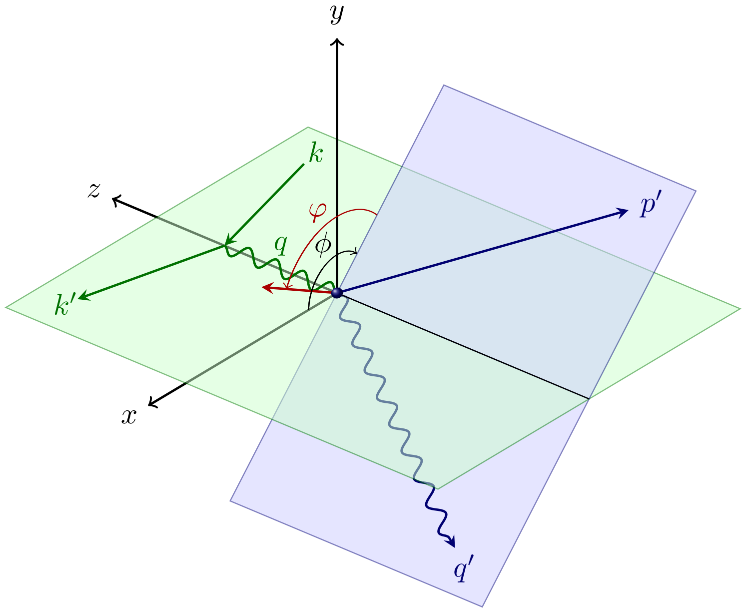

Figure 7.3.: DVCS in the target rest frame: The lepton scattering plane (green, spanned by k and k') intersects the nucleon scattering plane (blue, spanned by p' and q') under the angle ϕ. The red arrow is an example of a fully transverse nucleon spin (the polar angle between the transverse spin and the z-axis is $\theta = \pi/2$).

7.3. Impact of power corrections in DVCS

7.3.1. Preliminaries

Accessing the DVCS (sub-) process is achieved through lepton nucleon scattering with a real final state photon:

$$e^{\pm}(k, \lambda) + N(p, s) \to e^{\pm}(k', \lambda') + N(p', s') + \gamma(q', h') \, . \tag{7.32}$$

Here k, p (k', p', q') stand for the momenta and λ, s (λ', s', h') for the helicities of the corresponding particle in the initial (final) state. The amplitude $\mathcal{T}$ for this reaction is given by the coherent sum of the DVCS amplitude and the *Bethe-Heitler* (BH) amplitude,

$$\mathcal{T} = \mathcal{T}^{\mathrm{DVCS}} + \mathcal{T}^{\mathrm{BH}} \tag{7.33}$$

The latter arises from Feynman diagrams where the final state photon is emitted from the lepton line. It is given in terms of bilinear combinations of the usual Dirac and Pauli form factors F_1 and F_2. For the following numerical estimates we adopt the results from Kelly [61] for $F_{1,2}$.

In Ref. [45] the target rest frame, see Fig. 7.3, is utilized to express the cross section $d\sigma$. It is differential in x_B, Q^2, $|t|$ and two angles ϕ (angle between the lepton (green) and proton (blue) scattering plane in Fig. 7.3) and φ, the azimuthal angle (red) of the transverse nucleon polarization. Taking into account phase space factors BMJ write at leading order in QED

$$d\sigma = \frac{\alpha_{\rm em}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2 dx_B dQ^2 d|t| d\phi d\varphi\,, \tag{7.34}$$

where $\alpha_{\rm em} = e^2/(4\pi)$ and $y = (pq)/(pk)$. Assuming that the lepton is massless and with the help of the definition of x_B in Eq. (4.14) and the center-of-mass energy

$$s = (p+k)^2 \tag{7.35}$$

one can express y as

$$y = \frac{Q^2}{x_B(s-m^2)}\,. \tag{7.36}$$

Taking into account that the BH amplitude is real, one can write

$$|\mathcal{T}|^2 = \left|\mathcal{T}^{\rm BH}\right|^2 \pm \mathcal{I} + \left|\mathcal{T}^{\rm DVCS}\right|^2, \tag{7.37}$$

where $\mathcal{I}$ is the interference term

$$\mathcal{I} = 2\mathcal{T}^{\rm BH}\,{\rm Re}\,\mathcal{T}^{\rm DVCS}\,, \tag{7.38}$$

which enters in Eq. (7.37) with $+$ for electron and $-$ for positron scattering. The cross section $d\sigma$ (and each individual term in Eq. (7.37)) can be written as a sum over different target polarizations

$$d\sigma = d\sigma_{\rm unp}(\phi) + d\sigma_{\rm LP}(\phi)\cos\theta + \big[d\sigma_{\rm TP+}(\phi)\cos\varphi + d\sigma_{\rm TP-}(\phi)\sin\varphi\big]\sin\theta\,, \tag{7.39}$$

where the terms on the r.h.s. represent contributions from unpolarized, longitudinally polarized and two independent options of transversely polarized nucleons respectively (from left to right). θ is the angle between the nucleon spin and the z-axis. More details can be found in [45].

7.3.2. Fixed target, unpolarized

7.3.2.1. Beam spin sum/difference

Cross sections for unpolarized protons were recorded by the HALL A collaboration at Jefferson lab [62]. The experiment was performed using polarized electrons and is typically presented in terms of two linear combinations, the *beam spin sum*

$$d\Sigma_{\rm BS}\sigma = \frac{1}{2}\big[d\sigma_\lambda + d\sigma_{-\lambda}\big] \tag{7.40}$$

and the *beam spin difference*

$$d\Delta_{\rm BS}\sigma = \frac{1}{2}\big[d\sigma_\lambda - d\sigma_{-\lambda}\big]\,. \tag{7.41}$$

Here and below we indicate "discrete" variables, in this case the electron polarization λ, as subscripts on $d\sigma$, leaving the dependence on the other process variables implicit. As a function of ϕ, the beam spin sum $d\Sigma_{\rm BS}\sigma$ is even, whereas $d\Delta_{\rm BS}\sigma$ is odd, and they can be

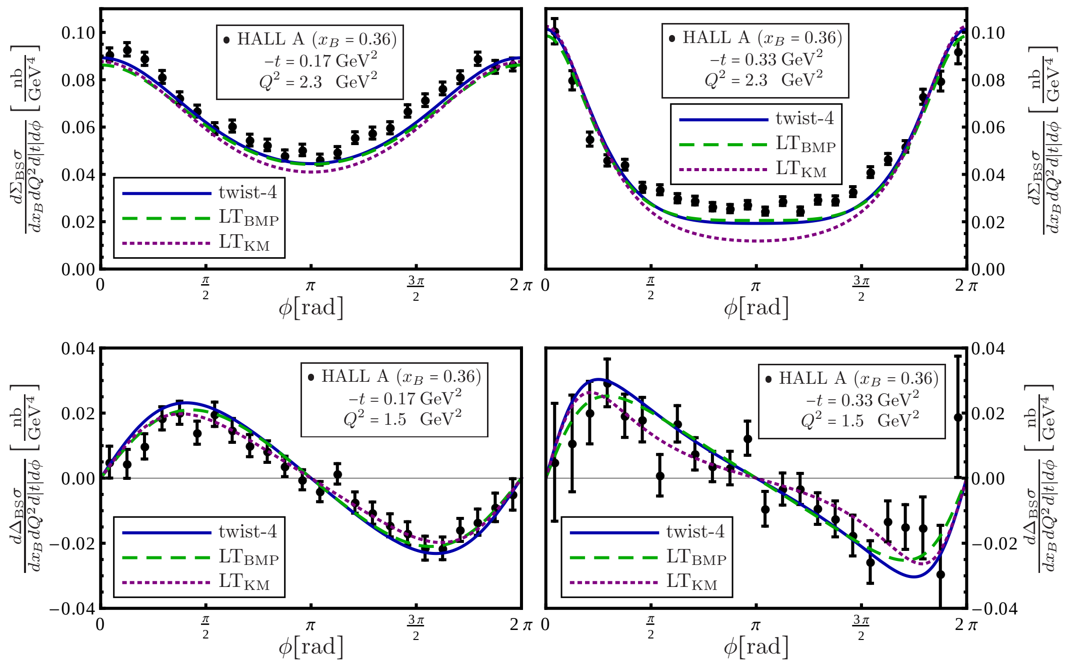

Figure 7.4.: Beam spin sum/difference measured by the Hall A collaboration [62] (black circles) in proton-electron collisions. The blue solid, green dashed, purple dotted curves correspond to predictions from twist-4, $\mathrm{LT_{BMP}}$, $\mathrm{LT_{KM}}$ approximations respectively using the GPD model *GK12* [51] and proton form factors from Kelly [61]. The angle ϕ is in the Trento convention.

written as a sum over Fourier harmonics [45,63]

$$
\begin{aligned}
&\frac{d\Sigma_{\mathrm{BS}}\sigma}{dx_B dQ^2 d|t| d\phi d\varphi} = \\
&\quad = \frac{\alpha_{\mathrm{em}}^3 x_B \sum_{n=0}^{2} c_{n,\mathrm{unp}}^{\mathrm{DVCS}} \cos(n\phi)}{8\pi Q^6 \sqrt{1+\gamma^2}} + \frac{\alpha_{\mathrm{em}}^3 \sum_{n=0}^{3} c_{n,\mathrm{unp}}^{\mathcal{I}} \cos(n\phi)}{8\pi y t Q^4 \sqrt{1+\gamma^2}\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} + \mathrm{BH}^2 , \\
&\frac{d\Delta_{\mathrm{BS}}\sigma}{dx_B dQ^2 d|t| d\phi d\varphi} = \\
&\quad = \frac{\alpha_{\mathrm{em}}^3 x_B\, s_{1,\mathrm{unp}}^{\mathrm{DVCS}} \sin(\phi)}{8\pi Q^6 \sqrt{1+\gamma^2}} + \frac{\alpha_{\mathrm{em}}^3 \sum_{n=1}^{2} s_{n,\mathrm{unp}}^{\mathcal{I}} \sin(n\phi)}{8\pi y t Q^4 \sqrt{1+\gamma^2}\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} .
\end{aligned}
\tag{7.42}
$$

“BH^2” stands for the contribution from the Bethe-Heitler process alone, which drops out in the beam spin difference. $\mathcal{P}_{1,2}(\phi)$ contain the ϕ-dependence of the electron propagators. Details can be found in [63]. Explicit expressions for the Fourier coefficients $c_{n,\mathrm{unp}}^{\mathrm{DVCS},\mathcal{I}}$, $s_{n,\mathrm{unp}}^{\mathrm{DVCS},\mathcal{I}}$ in terms of CFFs $\mathcal{F}_{a+}$ and $F_{1,2}$ are rather lengthy in full generality and can be found in [45].

Plugging the *GK12* model for the GPDs and Kelly's form factors into (7.42) along with the corresponding kinematical values [62] yields the predictions in Fig. 7.4. Note that the dependence on $\varphi \in [0, 2\pi]$ has been integrated out and the angle ϕ is taken in the so-called *Trento* convention, which is achieved by the substitution $\phi \to \pi - \phi$. We show the observables for the largest and smallest values for the momentum transfer and compare the

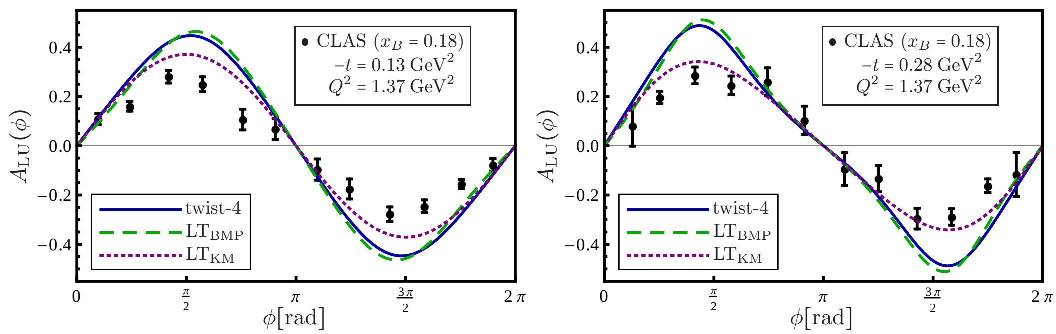

Figure 7.5.: Electron beam spin asymmetry $A_{\text{LU}}(\phi)$ measured by the CLAS collaboration [64] (black circles) for $-t = 0.13\,\text{GeV}^2$ (left panel) and $-t = 0.28\,\text{GeV}^2$ (right panel). The Trento convention is used for ϕ. The description of the curves is identical to Fig. 7.4.

three scenarios from Sec. 7.2.

For the observable $d\Sigma_{\text{BS}}\sigma$ at $Q^2 = 2.3\ \text{GeV}^2$ (upper two panels of Fig. 7.4) in the low-$|t|$ regime (upper left panel) all three conventions are compatible with each other (and the data). Going over to large t, the difference between LT_{BMP} (green dashed) and twist-4 (blue solid) curves remains negligible, whereas a clear deviation from LT_{KM} (purple dotted) becomes visible. It should be noted that the pure BH background makes about 95% of the cross section at $\phi = 0, 2\pi$ and ca. 50% at $\phi = \pi$. A close inspection reveals that the change in skewness $\xi_{\text{KM}} \to \xi_{\text{BMP}}$ makes almost no difference. The influence of the interference term is negligible and so are the double flip amplitudes $\mathcal{F}_{-+}$. A contribution of $|\mathcal{H}_{0+}|^2$ to $c^{\text{DVCS}}_{n=0,\text{unp}}$ is responsible for the "raise" of $d\Sigma_{\text{BS}}\sigma$ in the region $\phi \in (\pi/2, 3\pi/2)$. To some extent the excitation of $\mathcal{H}_{0+}$, which is not much smaller than $\mathcal{H}^{\text{KM}}_{++}$, see Fig. 7.1, was potentially expected to have some impact. This correction shifts the curves "in favor" of the high statistic data on $d\Sigma_{\text{BS}}\sigma$, whose description is generally regarded as a formidable task. The authors of [51] remarked, that the mismatch between theory and experiment may be related to $\text{Re}\,\mathcal{H}_{++}$, in particular to the absence of a D-term. Although this may be potentially true, our analysis points more towards the onset of $\mathcal{H}_{0+}$, which is neglected in [51].

The deviation of the three curves becomes more pronounced for $d\Delta_{\text{BS}}\sigma$ (Fig. 7.4, lower two panels), for which the value of $Q^2 = 1.5\ \text{GeV}^2$ is even lower, especially in the large-$|t|$ regime (lower right panel). This observable is inherently more sensitive to CFFs, since it is free of the $\left|\mathcal{T}^{\text{BH}}\right|^2$ term. It gets its biggest contribution from the interference term, which in turn is sensitive to $\text{Im}\{\mathcal{H}_{++}, \mathcal{E}_{++}, \widetilde{\mathcal{H}}_{++}\}$ at LO. Changing $\xi_{\text{KM}} \to \xi_{\text{BMP}}$ alone gives a net enhancement of about 15% for the $\sin(\phi)$ modulation, which is further increased by including the exact geometric factors and the twist-4 corrections. In parallel the influence of the CFFs $\mathcal{F}_{0+}$, especially $\mathcal{H}_{0+}$, induce a sizeable $\sin(2\phi)$ term. This modulation dampens the peak at $\phi \approx \frac{\pi}{4}$ but increases the magnitude of the curves in the middle region $\phi \in (\pi/2, 3\pi/2)$. The resulting $d\Delta_{\text{BS}}\sigma$ is then a superposition of the two effects.

7.3.2.2. Asymmetries

Taking the ratio between the beam spin difference and sum defines the dimensionless *single beam spin asymmetry*

$$A_{\mathrm{LU}}(\phi) = \frac{d\sigma_\lambda - d\sigma_{-\lambda}}{d\sigma_\lambda + d\sigma_{-\lambda}}\,. \tag{7.43}$$

Data was collected by the CLAS collaboration at Jefferson Lab and presented in [64]. We compare the predictions for very low $Q^2 = 1.37\,\mathrm{GeV}^2$ and two values of the invariant momentum transfer $-t \in \{0.13\,\mathrm{GeV}^2, 0.28\,\mathrm{GeV}^2\}$ in Fig. 7.5. The twist-4 and $\mathrm{LT_{BMP}}$ framework are again very close, see also Fig. 7.4, and give generally bigger asymmetries than $\mathrm{LT_{KM}}$ in the regions $\phi \sim \frac{\pi}{2}$ and $\phi \sim \frac{3\pi}{2}$ (in Trento convention). Although the denominator in A_{LU}, see Eq. (7.43), is typically larger for the former two approaches, this is not so pronounced in the aforementioned region of ϕ, where the differences in the numerator are numerically more important, cf. Fig. 7.4. Division by the beam spin sum has the effect that the asymmetry has its peak close to $\phi \simeq \pi/2$, where the beam spin differences of twist-4 and $\mathrm{LT_{BMP}}$ are approximately equal.

If a DVCS experiment can be performed with electron and position beams one can exploit the fact that the interference is sensitive to the sign of lepton charge and define the *charge-odd beam spin asymmetry*

$$A_{\mathrm{LU},\mathcal{I}}(\phi) = \frac{\left(d\sigma_\lambda^{e^+}(\phi) - d\sigma_{-\lambda}^{e^+}(\phi)\right) - \left(d\sigma_\lambda^{e^-}(\phi) - d\sigma_{-\lambda}^{e^-}(\phi)\right)}{d\sigma_\lambda^{e^+}(\phi) + d\sigma_{-\lambda}^{e^+}(\phi) + d\sigma_\lambda^{e^-}(\phi) + d\sigma_{-\lambda}^{e^-}(\phi)}\,, \tag{7.44}$$

where the superscript $e^\pm$ stands for positron $(+)$ and electron $(-)$ scattering. $A_{\mathrm{LU},\mathcal{I}}(\phi)$ has the advantage of separating the interference term. The numerator in Eq. (7.44) depends only on the (unpolarized) $\sin(n\phi)$ harmonics of $\mathcal{I}$, while the denominator is independent of $\mathcal{I}$. Access to the (unpolarized) $\cos(n\phi)$ harmonics of $\mathcal{I}$ is possible through the *beam charge asymmetry*

$$A_{\mathrm{C}}(\phi) = \frac{\left(d\sigma_\lambda^{e^+}(\phi) + d\sigma_{-\lambda}^{e^+}(\phi)\right) - \left(d\sigma_\lambda^{e^-}(\phi) + d\sigma_{-\lambda}^{e^-}(\phi)\right)}{d\sigma_\lambda^{e^+}(\phi) + d\sigma_{-\lambda}^{e^+}(\phi) + d\sigma_\lambda^{e^-}(\phi) + d\sigma_{-\lambda}^{e^-}(\phi)}\,. \tag{7.45}$$

Typically one considers Fourier coefficients of these asymmetries, which are defined as

$$\begin{aligned} A_{\mathrm{LU},\mathcal{I}}^{\sin(n\phi)} &= \frac{1}{\pi}\int_{-\pi}^{\pi} d\phi\, \sin(n\phi) A_{\mathrm{LU},\mathcal{I}}(\phi)\,, \\ (1+\delta_{n,0}) A_{\mathrm{C}}^{\cos(n\phi)} &= \frac{1}{\pi}\int_{-\pi}^{\pi} d\phi\, \cos(n\phi) A_{C}(\phi)\,. \end{aligned} \tag{7.46}$$

They are however not in direct correspondence with $s_{n,\mathrm{unp}}^{\mathcal{I}}$ and $c_{n,\mathrm{unp}}^{\mathcal{I}}$ due to a contamination by the ϕ-dependent denominators in Eqs. (7.44) and (7.45). $A_{\mathrm{LU},\mathcal{I}}^{\sin(n\phi)}$ and $A_{\mathrm{C}}^{\cos(n\phi)}$ has been measured by the HERMES collaboration at DESY [65]. The zeroth and first harmonics were reported to be significant, higher harmonics were consistent with zero. In Fig. 7.6 we present the results for the first harmonics, $A_{\mathrm{LU},\mathcal{I}}^{\sin(\phi)}$ (left panel) and $A_{\mathrm{C}}^{\cos(\phi)}$ (right panel). We selected data from an intermediate range of $x_B \sim 0.1$. The correlated values $x_B(t)$ and $Q^2(t)$ were interpolated and the quoted value of $Q^2 \sim 2.6\ \mathrm{GeV}^2$ is the average. Similar as in Fig. 7.5, the $\mathrm{LT_{BMP}}$ and twist-4 predictions for $A_{\mathrm{LU},\mathcal{I}}^{\sin(\phi)}$ are very close and their magnitude is considerably larger than $\mathrm{LT_{KM}}$. This can also be understood from Fig. 7.5, which is mostly

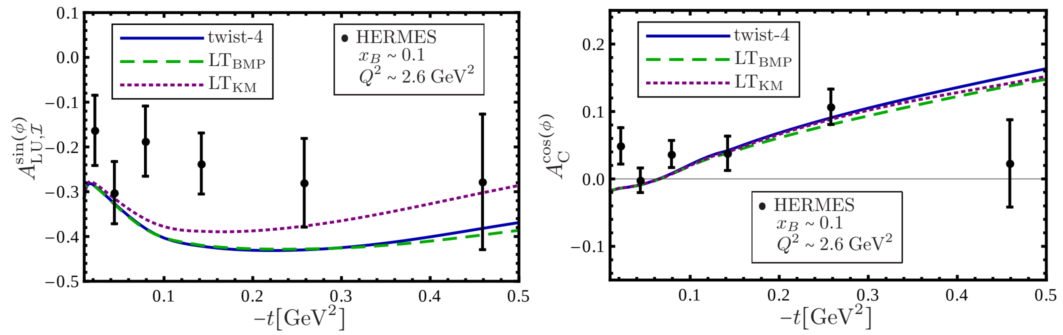

Figure 7.6.: The $\sin(\phi)$ harmonic of the charge-odd beam spin asymmetry (left panel) and the $\cos(\phi)$ harmonic of the beam charge asymmetry (right panel) vs $-t$. The description of the curves is identical to Fig. 7.4. Black circles denote data from the HERMES collaboration [65]. Statistical and systematical errors were added in quadrature.

dominated by the interference. Note that the negative sign comes from the definition (7.44), where the electron cross section is subtracted from the positron one. The situation is somewhat different in the case of $A_{\mathrm{C}}^{\cos(\phi)}$, where we observe that all three predictions do not differ much over the range of t under consideration. This may be coincidental, in any case it is special to the first harmonic. Other Fourier coefficients (for $n = 0, 2, \dots$) exhibit much larger deviations from framework to framework, see also the COMPASS-II estimates below. It seems that although the individual harmonics in the numerator of Eq. (7.45) have quite different coefficients for $\mathrm{LT_{KM}}$, $\mathrm{LT_{BMP}}$ and twist-4, the pollution by the ϕ-dependent denominator leads to a cancellation in the total Fourier coefficient. At the present stage it is not clear if $A_{\mathrm{C}}^{\cos(\phi)}$ is genuinely robust against higher twist corrections or if that statement is model specific. Finally note that both asymmetries go to zero at the kinematical boundary $|t| = |t_{\min}| \approx 0.008\ \mathrm{GeV}^2$ (very steeply), which is outside the plot region in Fig. 7.6.

The COMPASS-II experiment at CERN had a pilot DVCS run in November 2012 and further data acquisition is planned for the future. Given access to a 160 GeV beam of muons and antimuons with opposite polarizations, it was proposed to measure the charge spin asymmetry $A_{\mathrm{CS,U}}$. It is defined as

$$A_{\mathrm{CS,U}}(\phi) = \frac{d\sigma_{\lambda}^{\mu^+}(\phi) - d\sigma_{-\lambda}^{\mu^-}(\phi)}{d\sigma_{\lambda}^{\mu^+}(\phi) + d\sigma_{-\lambda}^{\mu^-}(\phi)}\,, \tag{7.47}$$

which can equivalently be written as

$$A_{\mathrm{CS,U}}(\phi) = \frac{A_{\mathrm{LU,DVCS}}(\phi) + A_{\mathrm{C}}(\phi)}{1 + A_{\mathrm{LU},\mathcal{I}}(\phi)}\,, \tag{7.48}$$

where

$$A_{\mathrm{LU,DVCS}}(\phi) = \frac{\left(d\sigma_{\lambda}^{\mu^+}(\phi) - d\sigma_{-\lambda}^{\mu^+}(\phi)\right) + \left(d\sigma_{\lambda}^{\mu^-}(\phi) - d\sigma_{-\lambda}^{\mu^-}(\phi)\right)}{d\sigma_{\lambda}^{\mu^+}(\phi) + d\sigma_{-\lambda}^{\mu^+}(\phi) + d\sigma_{\lambda}^{\mu^-}(\phi) + d\sigma_{-\lambda}^{\mu^-}(\phi)}\,. \tag{7.49}$$

In Eq. (7.48) one can neglect $A_{\mathrm{LU,DVCS}}(\phi)$, which contains only polarized contributions. One example projection for $x_B = 0.05$, $Q^2 = 2\ \mathrm{GeV}^2$, $-t = 0.2\ \mathrm{GeV}^2$ is shown in Fig. 7.7.

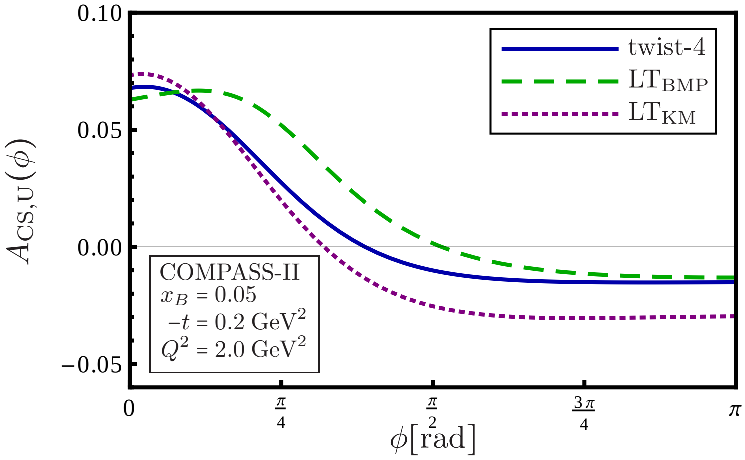

Figure 7.7.: Combined charge spin asymmetry for COMPASS-II kinematics. The description of the curves is identical to Fig. 7.4. The angle ϕ is in the Trento convention.

All three curves differ clearly, but could probably be compatible with experimental data. $A_{\mathrm{LU},\mathcal{I}}(\phi)$ does not differ much between $\mathrm{LT_{BMP}}$ and twist-4, see left panel of Fig. 7.6. Even the differences to $\mathrm{LT_{KM}}$ are not too large, see Fig. 7.5, compared with "1" in Eq. (7.48). Almost all differences arise from $A_{\mathrm{C}}(\phi)$. At $\phi \geq \frac{\pi}{2}$ the behavior is explained by the total cross section, see upper panels of Fig. 7.4, which enters in the denominator of A_{C} and depends quadratically (up to the pure BH term, which is less important here) on the CFFs. Since the numerator is linear in the CFFs, and $\mathrm{LT_{KM}}$ predicts smaller beam spin sums than the other two curves, the asymmetry A_{C} gets smaller in magnitude for the latter two at "large" ϕ. In the small-ϕ region the situation for the beam spin sum is reversed, $d\Sigma_{\mathrm{BS}}\sigma^{\mathrm{LT_{BMP}}} < d\Sigma_{\mathrm{BS}}\sigma^{\text{twist-4}} < d\Sigma_{\mathrm{BS}}\sigma^{\mathrm{LT_{KM}}}$, while in the intermediate ϕ range all three are approximately equal. The numerator for A_{C} depends on the real part of the CFFs. From the helicity conserving ones one gets the same contributions for all frameworks. However the nonconserving CFFs excite sizeable $\cos(\phi)$ and $\cos(2\phi)$ terms, largely dominated from $\mathcal{H}_{0+}$ and $\mathcal{E}_{0+}$. They show up visibly at $\phi \in (\pi/4, \pi/2)$ and are stronger for $\mathrm{LT_{BMP}}$, which is supported by Fig. 7.1. At small ϕ the differences are washed out by the aforementioned hierarchy of $d\Sigma_{\mathrm{BS}}$. Note that Fig. 7.7 does not contradict the right panel of Fig. 7.6, which refers only to a particular Fourier moment, whose stability was rather accidental. In fact, taking the first $\cos(\phi)$ moment of $A_{\mathrm{CS,U}}(\phi)$ averages out much of the discrepancy (max. 30%), while the zeroth and second moments differ up to factor four in magnitude. One should add that $A_{\mathrm{CS,U}}(\phi)$ exhibits also a strong dependence on the model details, whose differences are at least of the same order, see [51] for a comparison.

7.3.3. Fixed target, polarized

Measurements of cross sections with a longitudinally polarized proton target were pursued by the HERMES [66] and CLAS [67] collaborations with a positron and an electron beam respectively. Similarly to the (single) beam spin observables one defines the (longitudinal)

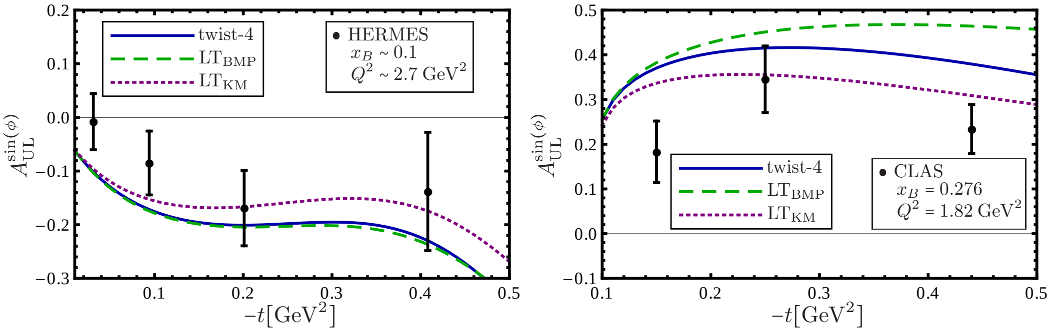

Figure 7.8.: Fourier moment $A_{\mathrm{UL}}^{\sin(\phi)}$ as a function of $-t$ measured by HERMES [66] (left panel) and CLAS [67] (right panel). Statistical and systematical errors were added in quadrature. The description of the curves is identical to Fig. 7.4.

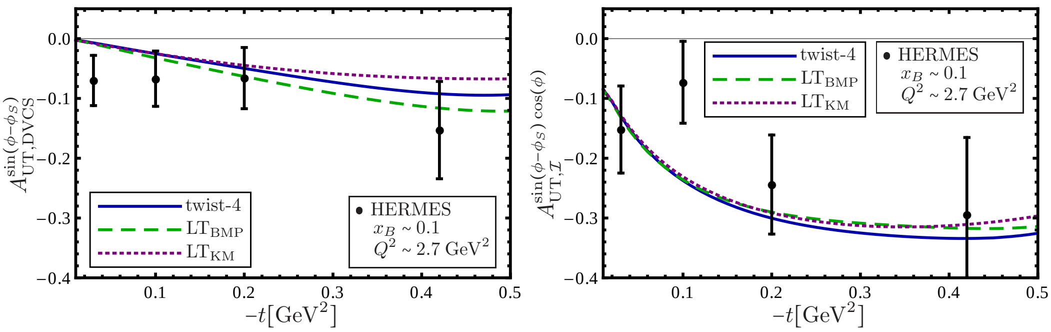

Figure 7.9.: Fourier moments $A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)}$ (left panel) and $A_{\mathrm{UT},\mathcal{I}}^{\sin(\phi-\phi_S)\cos(\phi)}$ (right panel) as a function of $-t$ measured by HERMES [68]. Statistical and systematical errors were added in quadrature. The description of the curves is identical to Fig. 7.4.

single target spin asymmetry

$$A_{\mathrm{UL}}(\phi) = \frac{d\sigma_s(\phi) - d\sigma_{-s}(\phi)}{d\sigma_s(\phi) + d\sigma_{-s}(\phi)} \quad (s \text{ longitudinal})\,. \tag{7.50}$$

It is an odd function of ϕ and mostly dominated by a $\sin(\phi)$ modulation, which motivates a study of the lowest Fourier coefficient

$$A_{\mathrm{UL}}^{\sin(\phi)} = \frac{1}{\pi}\int_{-\pi}^{\pi} d\phi\, \sin(\phi) A_{\mathrm{UL}}(\phi)\,. \tag{7.51}$$

The results are compiled in Fig. 7.8. For HERMES kinematics we see once more that the LT$_{\mathrm{BMP}}$ convention is capable of capturing most of the twist-4 features but differs significantly from LT$_{\mathrm{KM}}$. The mechanism is essentially the same as for A_{LU}. Going to even lower Q^2 as in the CLAS kinematics, the LT$_{\mathrm{BMP}}$ and twist-4 curves start to deviate, which is mostly related to the difference of $\mathcal{H}_{0+}$, $\widetilde{\mathcal{H}}_{a+}$, $\widetilde{\mathcal{E}}_{a+}$ for $a = +, 0$ between the two frameworks (although the sum $\widetilde{\mathcal{G}}_{a+}$ is fairly stable, see Fig. 7.2, it is not necessarily the case for the individual CFFs). The significantly larger value of x_B, compared to HERMES, further amplifies the differences.

Apart from the longitudinal polarization, HERMES also recorded events with transverse nucleon polarization [68]. In the spotlight were the *transverse target spin asymmetries*

$$\begin{aligned}
A_{\mathrm{UT,DVCS}}(\phi,\phi_S) &= \frac{1}{|s_\perp|}\frac{d\sigma^{e^+}(\phi,\phi_S)-d\sigma^{e^+}(\phi,\phi_S+\pi)+d\sigma^{e^-}(\phi,\phi_S)-d\sigma^{e^-}(\phi,\phi_S+\pi)}{d\sigma^{e^+}(\phi,\phi_S)+d\sigma^{e^+}(\phi,\phi_S+\pi)+d\sigma^{e^-}(\phi,\phi_S)+d\sigma^{e^-}(\phi,\phi_S+\pi)}\,,\\
A_{\mathrm{UT},\mathcal{I}}(\phi,\phi_S) &= \frac{1}{|s_\perp|}\frac{d\sigma^{e^+}(\phi,\phi_S)-d\sigma^{e^+}(\phi,\phi_S+\pi)-d\sigma^{e^-}(\phi,\phi_S)+d\sigma^{e^-}(\phi,\phi_S+\pi)}{d\sigma^{e^+}(\phi,\phi_S)+d\sigma^{e^+}(\phi,\phi_S+\pi)+d\sigma^{e^-}(\phi,\phi_S)+d\sigma^{e^-}(\phi,\phi_S+\pi)}\,.
\end{aligned} \tag{7.52}$$

Here the normalization $1/|s_\perp|$ removes the dependence on the average nucleon polarization $|s_\perp|$. The angle ϕ_S is related to those of Fig. 7.3 by $\varphi = \phi - \phi_S - \pi$. The numerators in the definition (7.52) are free of the BH cross sections, and the availability of positron and electron beams allows a separation of the interference and DVCS terms. As in the previous cases the reported measurements referred to Fourier coefficients

$$\begin{aligned}
(1+\delta_{n,0})A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)\cos(n\phi)} &= \int_{-\pi}^{\pi}\frac{d\phi\, d\phi_S}{(2\pi)^2}\,\cos(n\phi)\sin(\phi-\phi_S)A_{\mathrm{UT,DVCS}}(\phi,\phi_S)\,,\\
(1+\delta_{n,0})A_{\mathrm{UT},\mathcal{I}}^{\sin(\phi-\phi_S)\cos(n\phi)} &= \int_{-\pi}^{\pi}\frac{d\phi\, d\phi_S}{(2\pi)^2}\,\cos(n\phi)\sin(\phi-\phi_S)A_{\mathrm{UT},\mathcal{I}}(\phi,\phi_S)\,.
\end{aligned} \tag{7.53}$$

Analogous moments for coefficients involving $\sin(n\phi)\cos(\phi-\phi_S)$ were mostly compatible with zero [68]. Clear signals were obtained for the lowest values of n. Let us therefore focus on $A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)} \equiv A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)\cos(0\cdot\phi)}$ and $A_{\mathrm{UT},\mathcal{I}}^{\sin(\phi-\phi_S)\cos(\phi)}$, see Fig. 7.9. On the left panel $\mathrm{LT}_{\mathrm{KM}}$ gives the smallest (in absolute values) estimates for $A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)}$. Going over to $\mathrm{LT}_{\mathrm{BMP}}$ increases the asymmetry, while the remaining twist-4 contributions reduce it again. This is due to the onset (and subsequent reduction) of the helicity flip CFFs when going to $\mathrm{LT}_{\mathrm{BMP}}$ (twist-4). One has to keep in mind that although the curves give different predictions for the magnitude of $A_{\mathrm{UT,DVCS}}^{\sin(\phi-\phi_S)}$, they are largely compatible with the data. The $\cos(\phi)$ harmonic of the interference-type asymmetry $A_{\mathrm{UT},\mathcal{I}}^{\sin(\phi-\phi_S)\cos(\phi)}$, see right panel of Fig. 7.9, seems to be fairly stable against the power corrections. Here the situation is analogous to the charge asymmetry A_{C}, see Fig. 7.6. The immunity of $A_{\mathrm{UT},\mathcal{I}}^{\sin(\phi-\phi_S)\cos(\phi)}$ against higher order corrections seems to be rather accidental.

7.3.4. Collider experiments

In collider kinematics one typically deals with much higher photon virtualities (compared to fixed target experiments) and explores the phase space of small-x_B. Consequently the GPDs are probed in the small-ξ region, where they are rapidly growing functions with decreasing ξ, mediated through the small-ξ behavior of the sea quark contributions. In this limit the GPDs behave generically as

$$F^+(\xi,\xi) \sim \frac{\text{const.}}{\xi^\alpha}\,. \tag{7.54}$$

In the *GK12* model one has $1.0 \lesssim \alpha \lesssim 1.2$ for $F = H, E$ and $\alpha \lesssim 0.48$ for $F = \widetilde{H}$ and $\alpha \lesssim -0.48$ for $F = \widetilde{E}$. The dominant contribution in this region stems from $\left|\mathcal{T}^{\mathrm{DVCS}}\right|^2$ and

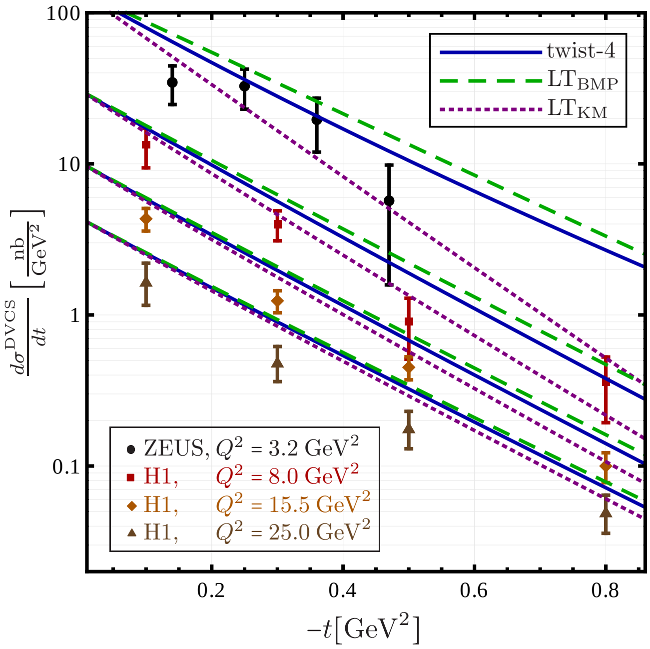

Figure 7.10.: DVCS cross section as measured by the H1 [69,70] and ZEUS [71] collaborations for different values of Q^2. The description of the curves is identical to Fig. 7.4.

one considers the *DVCS cross section*

$$\frac{d\sigma^{\mathrm{DVCS}}}{dt} = \frac{\alpha_{\mathrm{em}}^2 x_B^2 \left(1 - \frac{1-y-\frac{\gamma^2 y^2}{4}}{1-y+\frac{y^2}{2}+\frac{\gamma^2 y^2}{4}}\right)}{8\pi^2 Q^2 (1-x_B)\sqrt{1+\gamma^2}} \int_0^{2\pi} d\phi \int_0^{2\pi} d\varphi \, \frac{\left|\mathcal{T}^{\mathrm{DVCS}}\right|^2 + \mathcal{I}}{e^6}\,, \tag{7.55}$$

where the BH contribution has been explicitly subtracted. The angular integration removes all nonzero harmonics from $\left|\mathcal{T}^{\mathrm{DVCS}}\right|^2$ and suppresses contributions from the interference term. To a very good numerical accuracy this definition gives an observable uncontaminated by the BH process. In the limit $x_B \to 0$ and assuming that only the GPDs H and E are relevant (which is obviously true for the *GK12* model) the DVCS cross section can be approximated as [46]

$$\begin{aligned} \frac{d\sigma^{\mathrm{DVCS}}}{dt} \approx \frac{\pi\alpha_{\mathrm{em}}^2 x_B^2}{Q^4} \Bigg[&|\mathcal{H}_{++}|^2 - \frac{t}{4m^2}|\mathcal{E}_{++}|^2 + |\mathcal{H}_{-+}|^2 - \frac{t}{4m^2}|\mathcal{E}_{-+}|^2 \\ &+ \frac{1-y}{1-y-\frac{y^2}{2}}\left(|\mathcal{H}_{0+}|^2 - \frac{t}{4m^2}|\mathcal{E}_{0+}|^2\right)\Bigg]\,, \end{aligned} \tag{7.56}$$

which is occasionally referred to as the *Hand convention* [72].

The experiments H1 and ZEUS at HERA measured $d\sigma^{\mathrm{DVCS}}/dt$ [69–71], see Fig. 7.10. For the two data sets of $Q^2 = 3.2\ \mathrm{GeV}^2$ and $Q^2 = 8.0\ \mathrm{GeV}^2$ the model predictions differ enormously at large $-t$. The biggest effect can be explained by different skewness conventions: at small x_B one has $\xi_{\mathrm{BMP}} = (1 + t/Q^2)\xi_{\mathrm{KM}}$, see Eqs. (7.24) and (7.29). Also the contribution of $\mathcal{F}_{0+}$, being suppressed only by a factor of $\sqrt{-t/Q^2}$ at small-x_B, becomes sizeable at large $-t$, see Fig. 7.1 and Eq. (7.56). Both effects can accumulate up to roughly a factor six discrepancy between $\mathrm{LT}_{\mathrm{BMP}}$ and $\mathrm{LT}_{\mathrm{KM}}$, see Fig. 7.10. The rest of the twist-4 corrections has mainly the effect of reducing $\mathcal{F}_{0+}$ (see Fig. 7.1), which makes the net result for the cross section somewhat smaller again. A more detailed and analytical treatment of the small-x_B regime can be found in [46]. As expected, at larger Q^2 the three scenarios converge.

8. Conclusions and outlook

In this work we explored the finite-t and finite-m^2 effects in deeply virtual Compton scattering. These corrections can be identified as a subset of contributions of operators up to twist-4, which are derivatives of the leading twist-2 ones and do not require any additional nonperturbative input. The main new theoretical result is given by the time-ordered product of electromagnetic currents in momentum space $\mathcal{A}_{\mu\nu}$, accurate to the order $1/Q^2$ of the hard photon virtuality. We have given a concise presentation on how to obtain $\mathcal{A}_{\mu\nu}$ in the framework of the operator product expansion, using the recent progress on the separation techniques of the relevant leading twist descendants [23,24,42]. Most of the results of this thesis are published in [42,44,46].

Choosing a frame in which the light-cone is constructed out of the two participating photon momenta, we obtained a representation of $\mathcal{A}_{\mu\nu}$ in terms of GPDs, summarized in Eqs. (6.6)–(6.9) for a spin-$\frac{1}{2}$ particle and in Sec. 6.3 for a spin-0 target. Apart from the formulas involving GPDs, equivalent formulations in terms of double distributions and conformal moments are available. The results are translation and gauge invariant to the twist-5 accuracy. All appearing coefficient functions were found to be consistent with collinear factorization and the "new" coefficients have at most logarithmic singularities on the GPD cross-over line. In agreement with unitarity, dispersion relations (possibly subtracted) between imaginary and real parts have been shown to work. We explicitly demonstrated this on the level of convolutions, which can be extended to Compton form factors. Details can be found in Ref. [46].

To some extent the approach used here may be mapped to further processes. First and foremost, an extension to spin-1 hadrons, e.g. the deuteron, should be possible with moderate effort. It is conjectured by the author that this calculation is conceptually unproblematic and the techniques applied here generalize to the spin-1 sector. A second interesting topic would be to investigate more general kinematics, where the photon momentum q' is also virtual. This situation includes the usual DVCS process ($q'^2 \to 0$, $-q^2 \to \infty$), the *double DVCS* process [73] ($|q^2|, |q'^2| \to \infty$) and *timelike Compton scattering* [74] ($q^2 \to 0$, $q'^2 \to \infty$) as limiting cases. Factorization for the kinematical corrections may be established, if at least one hard scale is present. It remains an open but in principle straightforward problem to work out the details.

Focusing on DVCS with proton targets, which is the process that has been and is receiving the most attention, we continued to estimate the magnitude of the higher twist corrections. To this end we adopted a recent GPD parametrization [51] based on a double distribution ansatz, evaluated several representative observables in the framework of [45,63] and compared the impact of our results to two different leading twist approximations. The general conclusion is that the finite-Q^2 corrections cannot be neglected and should be taken into account in future data analyses.

In this context it is indispensable to note that any approximation truncated at some order in $1/Q^2$ is always convention dependent. It depends on the choice of reference frame, in particular on the selection of the light-cone directions used to classify large "plus" and small "minus" momenta as well as directions orthogonal to the thereby selected longitudinal plane. Associated with this convention is the choice of the polarization basis and the definition of a certain Compton form factor basis. Within a chosen convention one can specify e.g.

what is meant by a "leading twist approximation" (no helicity flips, etc.). Going over from one framework to another can be regarded as a Lorentz transformation to another reference frame and may lead to excitations of helicity flips. The relation between the Bjorken variable x_B and the skewness ξ, which enters amongst other places directly as arguments of the GPDs, is also affected by the freedom of choosing the light-cone. In different frameworks corresponding equations may differ by power-suppressed contributions. Although both effects are formally of order $1/Q$ in the Bjorken limit, there are examples where the numerical impact is significant. For example, changing the definition from the widely used $\xi = \xi_{\mathrm{KM}} = x_B/(2 - x_B)$ to $\xi = \xi_{\mathrm{BMP}}$ can induce large corrections in the small-x_B regime, relevant in collider experiments. Note that our twist-4 framework has to be seen as a convention as well. In particular the kinematical transformation to the CFF basis of Ref. [45], which was needed for the evaluation of cross sections, was treated exactly, i.e. untruncated in $1/Q$ and therefore contaminated by terms of order $1/Q^3$. The difference between keeping them and leaving them out can be used to estimate the remaining theoretical uncertainty and it was found to be mostly negligible. As a principal result, the twist-4 formalism removes the convention/frame dependence to the order $1/Q^2$ postponing further ambiguities to the $1/Q^3$ sector.

For certain observables it appears, that the two aforementioned "induced" corrections prove to be dominant, while the remaining higher twist contributions seem to be only a small addition. Exceptions to this rule were seen at very low Q^2 kinematics. It was found that for fixed target setups, the longitudinal-to-transverse CFF $\mathcal{H}_{0+}$ can have an important impact. In terms of the unpolarized total cross sections, where high statistic measurements are available, this is a desired effect while it increases the tension to other asymmetry observables. Fourier moments of asymmetries can have large corrections, however given the magnitude of the experimental error bars, they probably pose only rather loose constraints. As a rule of thumb one can conclude, that in the phase space region $Q^2 \gtrsim 4|t|$ the power corrections are under control and predictions are reliable.

At the moment, it is not entirely clear, to what extent our phenomenological conclusions depend on the details of the GPD model, especially at large x_B. In that domain the real part of CFFs and thus GPDs in the central region $|x| \leq |\xi|$ plays a more important role. On can expect a certain model dependence, given that the *GK12* parametrization neglects the possibility of a D-term. In order to clarify this point an extensive model study would be required, which goes far beyond the scope of this work.

For future analyses it is highly advisable to implement the finite-t and finite-m^2 effects in GPD fitting routines. It can be achieved at relatively small expense. For simple double distribution ansätze, like the *GK12* model, the Compton form factors and all quantities derived from them can be worked out analytically (though plagued by the heavy use of special functions). The corrections can be included as well in alternative approaches [47,48] employing Mellin-Barnes and dispersion relation techniques. The relevant formulas have been worked out in the appendix of Ref. [46] and in Ch. 6. In GPD fitting procedures one usually aims for a quark flavor separation. This can be done by including DVCS off a neutron and/or deeply virtual meson production. Analogous subleading twist contributions to the latter reaction channel are yet unknown and widely regarded as challenging. In parallel, GPD renormalization group effects and next-to-leading order QCD corrections should certainly be incorporated as well. At this order the gluon GPDs and the *transversity GPDs* directly enter in the amplitudes, see e.g. the reviews [15,16] for details. If one pursues the aim to really quantitatively "pin down" quark and gluon GPDs, all the above contributions should be taken into account. Especially for the extraction of a three-dimensional partonic image of the nucleon [19,20], it is required to cover a large range of the momentum transfer, where it is important to have an estimate of the t/Q^2 corrections.

A. The leading twist projector in practice

Recall the definition of the leading twist projector Π acting on a operator $\varphi(n)$, see Eq. (3.33),

$$[\Pi\varphi](x) = \sum_{k=0}^{\infty} \frac{(\bar{\partial}\bar{x}\partial)^k}{(k!)^2} \varphi\Big(n^\mu = \frac{1}{2}\lambda\sigma^\mu\bar{\lambda}\Big)\Big|_{\lambda=\bar{\lambda}=0} . \tag{A.1}$$

Converting the derivative operator to the usual vector formalism yields, cf. [23],

$$(\bar{\partial}\bar{x}\partial) = (x\partial_n) + (n\partial_n)(x\partial_n) - \frac{1}{2}(xn)\partial_n^2 . \tag{A.2}$$

Since this differential operator acting on $\varphi(n)$ is evaluated at $n = 0$ at the end, one can interchange it with $\varphi(n)$ under the replacement $n \leftrightarrow \partial_n$:

$$[\Pi\varphi](x) = \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \varphi(\partial_n)(xK)^k\Big|_{n=0} , \tag{A.3}$$

where

$$(xK) = (xn) + (xn)(n\partial_n) - \frac{1}{2}n^2(x\partial_n) . \tag{A.4}$$

Note that (xK) can be related to the generator of a special conformal transformation of a spinless field with scaling dimension one, see [23]. In this case the special conformal transformation is an inversion followed by a translation by $-x/2$ and another inversion. The k-th power of this generator is thus equal to

$$(xK)^k = \left(\frac{d}{d\theta}\right)^k \frac{1}{1-(xn)\theta + \theta^2 x^2 n^2/4}\Big|_{\theta=0} . \tag{A.5}$$

A proof for this equation can also be given directly by induction. Eq. (A.5) is a very useful result to deduce further properties and representations of Π. For example, the r.h.s. of (A.5) is equal, up to a prefactor of $k!\big(\sqrt{x^2n^2}/2\big)^k$, to the Gegenbauer polynomials $C_k^{(1)}\big(\frac{xn}{\sqrt{x^2n^2}}\big)$. Moreover, as an immediate byproduct of Eq. (A.5), one can see by direct calculation that $\partial_x^2(xK)^k$ vanishes, which translates into

$$\partial_x^2 \Pi(x,\lambda) = 0 , \tag{A.6}$$

i.e. Π generates operators that satisfy the d'Alembert equation. On the level of local operators this is equivalent to a property we already anticipated, namely the tracelessness. Eq. (A.5) also constitutes a good starting point for an expansion in x^2. One obtains

$$[\Pi\varphi](x) = \varphi(\partial_n) \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left(\frac{d}{d\theta}\right)^k \left[\frac{1}{1-(xn)\theta} - \frac{1}{4}\frac{\theta^2x^2n^2}{(1-(xn)\theta)^2}\right]\Bigg|_{\substack{n=0\\ \theta=0}} + \mathcal{O}(x^4) . \tag{A.7}$$

The k-fold derivative on the first term gives simply $k!$. In the second expression the θ^2-term in the numerator has to be "eaten up" by two derivatives, since we put $\theta = 0$ in the end. There are $k!/(2!(k-2)!)$ possibilities to do so. Then one arrives at $\mathcal{O}(x^2)$

$$[\Pi\varphi](x) = \varphi(\partial_n)e^{xn}\Big|_{n=0} - \frac{1}{4}x^2\varphi(\partial_n)\sum_{k=2}^{\infty}\frac{1}{k!(k-2)!}\left(\frac{d}{d\theta}\right)^{k-2}\frac{n^2}{(1-(xn)\theta)^2}\Bigg]\Bigg|_{\substack{n=0\\ \theta=0}} . \tag{A.8}$$

Evaluating the remaining θ-derivatives is now straightforward and simply yields a factor of $(k-1)!(xn)^{k-2}$. Thus

$$[\Pi\varphi](x) = \varphi(\partial_n)e^{xn}\Big|_{n=0} - \frac{1}{4}x^2\varphi(\partial_n)\int_0^1 du\, u n^2 e^{u(xn)}\Big|_{n=0} . \tag{A.9}$$

We can now reverse the "trick" from above, interchanging $n \leftrightarrow \partial_n$, which gives

$$[\Pi\varphi](x) = e^{x\partial_n}\varphi(n)\Big|_{n=0} - \frac{1}{4}x^2\int_0^1 du\, u\,\partial_n^2\, e^{u(x\partial_n)}\varphi(n)\Big|_{n=0} . \tag{A.10}$$

Since $e^{u(x\partial_n)}$ is nothing else than the translation operator, that shifts functions of n to $n+ux$, we obtain the final expression

$$[\Pi\varphi](x) = \varphi(x) - \frac{1}{4}x^2\int_0^1 du\, u[\partial^2\varphi](ux) + \mathcal{O}(x^4) . \tag{A.11}$$

Eq. (A.11) will be a very convenient form of Π for the calculation of the helicity amplitudes to the twist-4 approximation.

There are many useful identities involving Π, see [23] for details. Here we prove one of them, which is necessary to verify translation invariance, namely the "product rule"

$$\Pi(x,\lambda)\lambda^\alpha\bar\lambda^{\dot\alpha}\varphi(\lambda,\bar\lambda) = \bar x^{\dot\alpha\alpha}[\Pi\varphi](x) - \frac{1}{2}x^2\bar\partial^{\dot\alpha\alpha}\int_0^1 du\,[\Pi\varphi](ux) . \tag{A.12}$$

To see this, we start by using the very definition of Π in Eq. (A.1). Since λ and $\bar\lambda$ are put to zero at the end, the factor $\lambda^\alpha\bar\lambda^{\dot\alpha}$ has to be "caught" by the derivatives $(\bar\partial\bar x\partial)$. This can happen if both derivatives from one factor $(\bar\partial\bar x\partial)$ or if derivatives from two different factors "hit" the expression $\lambda^\alpha\bar\lambda^{\dot\alpha}$. Out of $(\bar\partial\bar x\partial)^k$ there are k and $k(k-1)$ possibilities in the two respective cases. Then we can write

$$\Pi(x,\lambda)\lambda^\alpha\bar\lambda^{\dot\alpha}\varphi(n) = \sum_{k=1}^{\infty}\frac{(\bar\partial\bar x\partial)^{k-1}}{k!(k-1)!}\bar x^{\dot\alpha\alpha}\varphi(n) + \sum_{k=2}^{\infty}\frac{(\bar\partial\bar x\partial)^{k-2}}{k!(k-2)!}\bar x^{\dot\beta\alpha}\bar x^{\dot\alpha\beta}\frac{\partial}{\partial\bar\lambda^{\dot\beta}}\frac{\partial}{\partial\lambda^\beta}\varphi(n) . \tag{A.13}$$

In the second term we represent

$$\frac{\partial}{\partial\bar\lambda^{\dot\beta}}\frac{\partial}{\partial\lambda^\beta} = \frac{1}{2}\partial_{\beta\dot\beta}(\bar\partial\bar x\partial) , \tag{A.14}$$

see Eq. (2.16). The differential operator $\partial_{\beta\dot\beta} = (\sigma^\mu)_{\beta\dot\beta}\partial_{x_\mu}$ can be pulled out of the sum in Eq. (A.13) by noting $\partial_{\beta\dot\beta}(\bar\partial\bar x\partial)^{k-1} = (k-1)(\bar\partial\bar x\partial)^{k-2}\partial_{\beta\dot\beta}(\bar\partial\bar x\partial)$. Then both sums in

Eq. (A.13) have the same structure again,

$$\Pi(x,\lambda)\lambda^{\alpha}\bar{\lambda}^{\dot{\alpha}}\varphi(n) = \left(\bar{x}^{\dot{\alpha}\alpha} + \frac{1}{2}\bar{x}^{\dot{\beta}\alpha}\bar{x}^{\dot{\alpha}\beta}\partial_{\beta\dot{\beta}}\right)\sum_{k=1}^{\infty}\frac{(\bar{\partial}\bar{x}\partial)^{k-1}}{k!(k-1)!}\varphi(\boldsymbol{n})\,. \tag{A.15}$$

Shifting the summation in k by -1 and expressing

$$\frac{1}{k+1} = \int_0^1 du\, u^k \tag{A.16}$$

one easily sees

$$\Pi(x,\lambda)\lambda^{\alpha}\bar{\lambda}^{\dot{\alpha}}\varphi(\boldsymbol{n}) = \left(\bar{x}^{\dot{\alpha}\alpha} + \frac{1}{2}\bar{x}^{\dot{\beta}\alpha}\bar{x}^{\dot{\alpha}\beta}\partial_{\beta\dot{\beta}}\right)\int_0^1 du\,[\Pi\varphi](ux)\,. \tag{A.17}$$

Furthermore, recall from the spinor formalism that

$$\bar{x}^{\dot{\beta}\alpha}\bar{x}^{\dot{\alpha}\beta}\partial_{\beta\dot{\beta}} = x_{\mu}x_{\nu}\partial_{\rho}(\bar{\sigma}^{\mu}\sigma^{\rho}\bar{\sigma}^{\nu})^{\dot{\alpha}\alpha}\,. \tag{A.18}$$

By employing the "anticommutation" relations

$$(\bar{\sigma}^{\mu}\sigma^{\nu})^{\dot{\alpha}}{}_{\dot{\beta}} + (\bar{\sigma}^{\nu}\sigma^{\mu})^{\dot{\alpha}}{}_{\dot{\beta}} = 2g^{\mu\nu}\delta^{\dot{\alpha}}{}_{\dot{\beta}} \tag{A.19}$$

it can be shown

$$\Pi(x,\lambda)\lambda^{\alpha}\bar{\lambda}^{\dot{\alpha}}\varphi(n) = \left(\bar{x}^{\dot{\alpha}\alpha}\big(1+(x\partial)\big) - \frac{1}{2}x^2\partial^{\dot{\alpha}\alpha}\right)\int_0^1 du\,[\Pi\varphi](ux)\,. \tag{A.20}$$

Next, we can replace $1+(x\partial) \to \partial_u u$ under the u-integral and subsequent direct integration completes the proof of (A.12).

B. Fourier transformation cheat sheet

In this appendix we compile (and prove) several Fourier integrals needed for the computation of the DVCS amplitudes. The basic integrals with quartic denominator in x read

$$\mu^{4-d}\int\frac{d^dx}{\pi^2}\,e^{-iqx}\frac{1}{(x^2-i0)^2}=\frac{2i}{4-d}+i\left(\ln\left(\frac{-q^2-i0}{\mu^2}\right)+\gamma_E-\ln(4\pi)\right)+\mathcal{O}(4-d)\,, \tag{B.1}$$

$$\int\frac{d^4x}{\pi^2}\,e^{-iqx}\frac{x_\mu}{(x^2-i0)^2}=\frac{-2q_\mu}{q^2+i0}\,, \tag{B.2}$$

$$\int\frac{d^4x}{\pi^2}\,e^{-iqx}\frac{x_\mu x_\nu}{(x^2-i0)^2}=-2i\left(\frac{g_{\mu\nu}}{q^2+i0}-\frac{2q_\mu q_\nu}{(q^2+i0)^2}\right), \tag{B.3}$$

$$\int\frac{d^4x}{\pi^2}\,e^{-iqx}\frac{x_\mu x_\nu x_\rho}{(x^2-i0)^2}=-4\left(\frac{g_{\mu\nu}q_\rho+g_{\mu\rho}q_\nu+g_{\nu\rho}q_\mu}{(q^2+i0)^2}-4\frac{q_\mu q_\nu q_\rho}{(q^2+i0)^3}\right). \tag{B.4}$$

In the first line (B.1) the integral is written in d dimensions and diverges for $d=4$. The associated mass scale is μ. This is not a problem, since only derivatives of it contribute in the calculation. These are all well-defined in four dimensions and therefore unproblematic.

Further one needs a couple of basic integrals with quadratic denominator in x:

$$\int\frac{d^4x}{\pi^2}\,e^{-iqx}\frac{1}{x^2-i0}=\frac{-4i}{q^2+i0}\,, \tag{B.5}$$

$$\int\frac{d^4x}{\pi^2}\,e^{-iqx}\frac{x_\mu}{x^2-i0}=\frac{-8q_\mu}{(q^2+i0)^2}\,, \tag{B.6}$$

$$\int\frac{d^4x}{\pi^2}\,e^{-iqx}\frac{x_\mu x_\nu}{x^2-i0}=-8i\left(\frac{g_{\mu\nu}}{(q^2+i0)^2}-4\frac{q_\mu q_\nu}{(q^2+i0)^3}\right), \tag{B.7}$$

$$\int\frac{d^4x}{\pi^2}e^{-iqx}\frac{x_\mu x_\nu x_\rho}{x^2-i0}=-32\left(\frac{g_{\mu\nu}q_\rho+g_{\mu\rho}q_\nu+g_{\nu\rho}q_\mu}{(q^2+i0)^3}-6\frac{q_\mu q_\nu q_\rho}{(q^2+i0)^4}\right). \tag{B.8}$$

It is sufficient to prove Eqs. (B.1) and (B.5), since all other formulas can be obtained by differentiating the former ones with respect to the Fourier variable q. Both (B.1) and (B.5) originate from a more general integral

$$I_r(q)=\int d^dx\,e^{-iqx}\frac{1}{(-x^2+i0)^r}\,. \tag{B.9}$$

By employing the *Schwinger parametrization*,

$$\frac{1}{(-x^2+i0)^r}=\frac{(-i)^r}{\Gamma(r)}\int_0^\infty du\,u^{r-1}e^{-iu(x^2-i0)}\,, \tag{B.10}$$

one can trade the denominator for an additional integral. The convergence of (B.10) is ensured by the "$i0$" prescription. Now combine the exponential with the one from the Fourier transformation and complete the square. Then shift the d^dx-integration by $x\to x+q/(2u)$

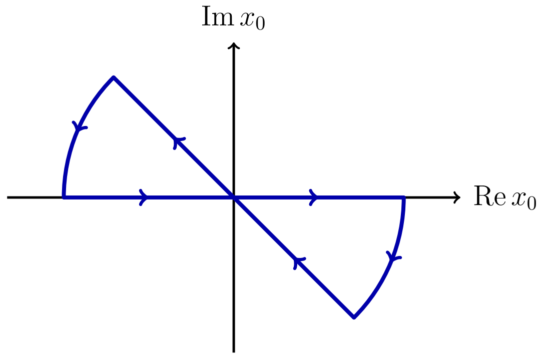

Figure B.1.: Integration path in the complex x_0-plane.

and obtain

$$I_r(q) = \frac{(-i)^r}{\Gamma(r)} \int_0^\infty du\, u^{r-1} e^{-iu\left(-q^2/(4u^2)-i0\right)} \int d^d x\, e^{-iux^2} \,. \tag{B.11}$$

An integral of the type

$$\int_{-\infty}^{\infty} dx_0\, e^{-iux_0^2} \tag{B.12}$$

can be evaluated using Cauchy's theorem. As the integrand is analytic in the whole complex x_0-plane, the closed contour integral in Fig. B.1 vanishes. Now choose the "tilting angle" to be $-\pi/4$. Then the contributions of the arcs at infinity vanish and (B.12), being equal to the integral on the "tilted" line, is essentially Gaussian:

$$\int_{-\infty}^{\infty} dx_0\, e^{-iux_0^2} = e^{-i\frac{\pi}{4}} \sqrt{\frac{\pi}{u}} \,. \tag{B.13}$$

The result for the spatial integrations is given by the complex conjugate of the above.

Thus

$$I_r(q) = \frac{\pi^{\frac{d}{2}} e^{i\frac{\pi}{4}(d-2-2r)}}{\Gamma(r)} \int_0^\infty du\, u^{r-1-d/2} e^{-iu\left(-q^2/(4u^2)-i0\right)} \,. \tag{B.14}$$

Finally, the remaining integration is elemental and gives

$$I_r(q) = -i\pi^{\frac{d}{2}} \frac{\Gamma(d/2-r)}{\Gamma(r)} \left(-\frac{q^2+i0}{4}\right)^{r-d/2} \,. \tag{B.15}$$

Putting $d \to 4$ and $r \to 1$ immediately establishes (B.5). For $d = 4$ and $r = 2$ the result is ill-defined, the relevant terms of the Laurent series in $4-d$ is given in (B.1).

C. From DDs to GPDs

In this appendix we address the issue of converting convolutions of coefficient functions with double distributions into a representation involving generalized parton distributions.

In the easiest scenario, where the coefficient function is unity, one simply needs to insert the DD parametrization of Eq. (4.61) (with $z_1 = -z_2 = \frac{1}{2}$) into the GPD-parametrization in Eq. (4.58). By comparing independent Lorentz structures, cf. Eq. (4.60), it can be easily seen that (skipping additional arguments t, μ^2, which are not important here)

$$\begin{aligned} &\int d\beta d\alpha\, \delta(x-\beta-\alpha\xi) h(\beta,\alpha) = H(x,\xi) + E(x,\xi) \equiv M(x,\xi)\,, \\ &\int d\beta d\alpha\, \delta(x-\beta-\alpha\xi) \tilde{h}(\beta,\alpha) = \widetilde{H}(x,\xi) \end{aligned} \tag{C.1}$$

and

$$\begin{aligned} &\int d\beta d\alpha\, \delta(x-\beta-\alpha\xi) \Phi(\beta,\alpha) = -\partial_x E(x,\xi)\,, \\ &\int d\beta d\alpha\, \delta(x-\beta-\alpha\xi) \tilde{\Phi}(\beta,\alpha) = -\xi\partial_x \widetilde{E}(x,\xi)\,. \end{aligned} \tag{C.2}$$

Therefore, in order to obtain a GPD (we use that notion in a more general sense here, e.g. $-\partial_x E(x,\xi)$ is also denoted as a GPD), one needs to integrate along the line $x-\beta-\alpha\xi = 0$ in the (β,α)-plane, see Fig. 4.2. In the physical region $\xi \leq 1$, $|x| \leq 1$ the straight line traverses the square-shaped support region of the DD, intersecting the β-axis at x. The slope, being $-\frac{1}{\xi}$, is always steep, i.e. ≤ -1. For the GPD on a cross-over line $x = \pm\xi$, the line goes through the top (+) or the bottom (−) vertex. In the inner region, $|x| \leq \xi$, the line crosses the α-axis, whereas in the outer region, $|x| \geq \xi$, it does not. In the forward limit $\xi \to 0$ the line is parallel to the α-axis.

The convolutions appearing in the DD representation of the helicity amplitudes generically fall into one of the categories

$$\begin{aligned} \mathfrak{I}_n^{\Phi} &= \int d\beta d\alpha\, \beta^n f(\omega) \Phi(\beta,\alpha)\,, \\ \mathfrak{I}_n^{\tilde{\Phi}} &= \int d\beta d\alpha\, \beta^n f(\omega) \tilde{\Phi}(\beta,\alpha)\,, \\ \mathfrak{I}_n^{h} &= \int d\beta d\alpha\, \beta^n f(\omega) h(\beta,\alpha)\,, \\ \mathfrak{I}_n^{\tilde{h}} &= \int d\beta d\alpha\, \beta^n f(\omega) \tilde{h}(\beta,\alpha)\,, \end{aligned} \tag{C.3}$$

where $n = 0, 1, 2$ and f is an arbitrary function of the single argument $\omega = \frac{1}{2}\big(\frac{\beta}{\xi} + \alpha + 1\big)$.

For a start consider $\mathfrak{I}^{\Phi}_{n=0}$. By inserting unity

$$1 = \int dx\, \delta(x - \beta - \alpha\xi) \tag{C.4}$$

one gets

$$\mathfrak{I}^{\Phi}_{0} = -\int dx\, f\left(\frac{x+\xi}{2\xi}\right) \partial_x E(x,\xi)\,, \tag{C.5}$$

which, after integration by parts, turns into

$$\mathfrak{I}^{\Phi}_{0} = \frac{1}{2\xi} \int dx\, f'\left(\frac{x+\xi}{2\xi}\right) E^q(x,\xi)\,, \tag{C.6}$$

where f' denotes the derivative of f. Next, in the $\mathfrak{I}^{\Phi}_{n=1}$ case, we insert the δ-function from above, then we can replace $\beta \to (x - \alpha\xi)$ as well as

$$(x - \alpha\xi)\delta(x - \beta - \alpha\xi) = (x\partial_x + \xi\partial_\xi)\theta(x - \beta - \alpha\xi)\,. \tag{C.7}$$

The (β, α)-convolution with the Heaviside step-function θ from above gives

$$\int d\beta d\alpha\, \theta(x - \beta - \alpha\xi)\Phi(\beta,\alpha) = -E(x,\xi)\,. \tag{C.8}$$

The differential operator $x\partial_x$ can be shuffled onto the function f, where it can be equated to a derivative w.r.t. ξ by means of the obvious identity

$$x\partial_x\, f\left(\frac{x+\xi}{2\xi}\right) = -\xi\partial_\xi\, f\left(\frac{x+\xi}{2\xi}\right). \tag{C.9}$$

Thus it is possible to trade $x\partial_x + \xi\partial_\xi$ for a differential operator in ξ outside of the convolution integral:

$$\mathfrak{I}^{\Phi}_{1} = -\xi^2\partial_\xi\, \xi^{-1} \int dx\, f\left(\frac{x+\xi}{2\xi}\right) E(x,\xi)\,. \tag{C.10}$$

Last, in the case of $\mathfrak{I}^{\Phi}_{n=2}$ we represent

$$(x - \alpha\xi)^2\delta(x - \beta - \alpha\xi) = (x^2\partial_x + 2x\xi\partial_\xi)\theta(x - \beta - \alpha\xi) + \xi^2\partial^2_\xi \int_{-1}^{x} dx'\, \theta(x' - \beta - \alpha\xi)\,. \tag{C.11}$$

Let F be a primitive of f, i.e. $F'(\omega) = f(w)$ and thus $F'(\omega) = 2\xi\partial_x F(\omega) = -2x^{-1}\xi^2\partial_\xi F(\omega)$, then we can write

$$\mathfrak{I}^{\Phi}_{2} = -2\xi^2 \int dx \left\{ \left[\partial_\xi F\left(\frac{x+\xi}{2\xi}\right)\right](x\partial_x + 2\xi\partial_\xi) + F\left(\frac{x+\xi}{2\xi}\right)\xi\partial^2_\xi \right\} E(x,\xi)\,. \tag{C.12}$$

Now the term $\big[\partial_\xi F(\dots)\big]x\partial_x$ can be turned into $\big[\xi\partial^2_\xi F(\dots)\big]$ by integration by parts. Then we have brought the integral in the form of a double derivative

$$\mathfrak{I}_2^{\Phi} = -2\xi^3 \partial_\xi^2 \int dx\, F\!\left(\frac{x+\xi}{2\xi}\right) E(x,\xi)\,, \quad \text{with } F' = f\,. \tag{C.13}$$

Notice that we get the corresponding formulas for $\widetilde{E}$ by the replacement $\Phi \to \tilde{\Phi}$, $E \to \xi \widetilde{E}$, because of Eq. (C.2).

The identities involving convolutions with h are derived completely analogous, and read

$$\begin{aligned}
\mathfrak{I}_0^{h} &= \int dx\, f\!\left(\frac{x+\xi}{2\xi}\right) M(x\,\xi)\,,\\
\mathfrak{I}_1^{h} &= -2\xi^2 \partial_\xi \int dx\, F\!\left(\frac{x+\xi}{2\xi}\right) M(x\,\xi)\,, \quad \text{with } F' = f\,,\\
\mathfrak{I}_2^{h} &= 4\xi^2 \partial_\xi \xi^2 \partial_\xi \int dx\, G\!\left(\frac{x+\xi}{2\xi}\right) M(x\,\xi)\,, \quad \text{with } G'' = f\,.
\end{aligned} \tag{C.14}$$

Of course, the above equations generalize to $\tilde{h}$ by the replacement $h \to \tilde{h}$, $M \to \widetilde{H}$.

These identities establish the quoted result in Eq. (6.1), where the appearance of the signature-projected GPDs is a consequence of the (anti-)symmetrization (5.1) and the reflection properties (4.67).

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